Journal of Intelligent and Robotic Systems **36:** 331–347, 2003.

Model-Reference Fuzzy Adaptive Control as a Framework for Nonlinear System Control

IGOR ŠKRJANC, SAŠO BLAŽIČ and DRAGO MATKO
Faculty of Electrical Engineering, University of Ljubljana, Tržaška 25, 1000 Ljubljanam, Slovenia; e-mail: igor.skrjanc@fe.uni-lj.si

(Received: 28 November 2001; in final form: 15 July 2002)

Abstract. The paper presents a general methodology of adaptive control based on fuzzy model to deal with unknown plants. The problem of parameter estimation is solved using a direct approach, i.e. the controller parameters are adapted without explicitly estimating plant parameters. Thus, very simple adaptive and control laws are obtained using Lyapunov stability criterion. The generality of the approach is substantiated by Stone-Weierstrass theorem, which indicates that any continuous function can be approximated by fuzzy basis function expansion. In the sense of adaptive control, this implies the adaptive law with fuzzified adaptive control parameters. The proposed control algorithm may be viewed as an extension of classical adaptive control for linear plants, but compared to the latter it provides higher adaptation ability and consequently better performance if the plant is nonlinear. The global stability of the control system is assured and the tracking error converges to the residual set that depends on fuzzification properties. The main advantage of the approach is simplicity that suits control engineers since wide range of industrial processes can be controlled by the proposed method. In the paper, the control of heat exchanger is performed.

Key words: adaptive control, fuzzy control, fuzzy model, model-reference adaptive control.

1. Introduction

The nature of dynamical systems usually implies slow changes of systems parameters and changes of the parameters due to the different operating conditions or operating point. In that case an adaptive controller should be designed to follow the changes of operating conditions and adapt in certain prescribed way.

In recent years, a lot of effort has been put to neuro-fuzzy identification of complex plants, which can not be easily theoretically modelled. Based on neuro-fuzzy presentation of the plant dynamics the neuro-fuzzy adaptive control approaches appeared in literature [16] where detailed discussion on identification and control of dynamical systems based on neural networks is given, in [11] where the tracking performance of model reference adaptive control using multilayer neural networks based on Lyapunov stability approach is studied and in [23, 25] where stable adaptive fuzzy controller for nonlinear systems is designed and explained, in [15] an adaptive control using multiple models is developed and investigated. A direct adaptive fuzzy-model-based control algorithm is presented in [1]. In this case the controller is based on an inverse semi-linguistic fuzzy process model which is identified and adapted via input-matching technique using a general gradient-descent algorithm. An adaptive fuzzy sliding-mode controller and its application to a robot manipulator arm is presented in [2]. In [4], two methods for adaptation of nonlinear adaptive controllers are presented and compared, namely, the data-driven and the knowledge-based adaptation. Both approaches are compared by application to temperature control of a heat exchanger.

In our paper a general methodology of adaptive control based on fuzzy model to deal with plants with unknown parameters is presented. We introduce a novel model reference fuzzy adaptive control system which is based on the fuzzy basis function expansion. The generality of the proposed algorithm is substantiated by Stone–Weierstrass theorem which indicates that any continuous function can be approximated by fuzzy basis function expansion. The combination of adaptive control theory based on models obtained by fuzzy basis function expansion results in direct model-reference fuzzy adaptive control which provides higher adaptation ability than basic adaptive control systems. The proposed control algorithm is the extension of direct model-reference fuzzy adaptive control to nonlinear plants. Direct fuzzy adaptive controller directly adjusts the parameter of fuzzy controller to achieve approximate asymptotic tracking of the model-reference input. The main advantage of the proposed approach is simplicity together with high performance, and it has been shown that the closed-loop system using direct fuzzy adaptive controller is globally stable and the tracking error converges to the residual set which depends on fuzzification properties. The proposed approach can be implemented on a wide range of industrial processes. In the paper the foundation of the proposed algorithm are given and some simulation examples are shown and discussed.

The paper is organized as follows: in Section 2 the description of direct model-reference fuzzy adaptive system is given, in Section 3 we are introducing the model of nonlinear heat-exchanger plant which was used in our simulation study and in Section 4 the simulation study is described. In Section 5 some main observations are discussed.

2. Direct Model-Reference Fuzzy Adaptive Control

Model reference adaptive control systems are proven to be globally stable under certain assumptions on the unknown process: phase minimality; no disturbance or unmodelled dynamics; linearity; time invariance; and knowledge of the process relative degree and the sign of so-called high-frequency gain. Unfortunately, the assumptions given above are often violated in practice and “adaptive algorithms as published in the literature are likely to produce unstable control systems if they are implemented on physical systems directly as they appear in the literature” [22]. Many of the above assumptions can be circumvented on the cost of more complex adaptive and control laws. Robustness of the adaptive systems to unmodelled dynamics and bounded disturbance is treated in [14, 19]. Nonlinear adaptive control has been widely studied in the last decade [12] but the results obtained seem not to be easily transferred to the engineering society since they require fairly good knowledge of mathematics and thus these approaches are avoided by practicing engineers.

2.1. MODEL-REFERENCE ADAPTIVE CONTROL OF LTI SYSTEMS

The globally stable continuous model-reference adaptive control dynamic is given first. The goal of the model-reference adaptive system is to design a controller which forces the process to follow the model output, which is in the case of first order plant given by the following equation

$$G_m(s) = \frac{y_m(s)}{w(s)} = \frac{b_m}{s + a_m} \tag{1}$$

where $w(t)$ stands for the reference signal and $y_m(t)$ for reference-model output. To obtain perfect model following a pre-filter with gain f and the gain q in the feedback loop should be designed. Assuming the process transfer function

$$G_p(s) = \frac{y_p(s)}{u(s)} = \frac{b}{s + a} \tag{2}$$

and control law given in the following equation

$$u = fw - qy_p \tag{3}$$

the closed-loop transfer function is given by

$$G_w(s) = \frac{fb}{s + a + bq}. \tag{4}$$

The parameter errors between open-loop parameters and model-reference parameters are given next

$$\begin{aligned} \tilde{b} &= fb - b_m, \\ \tilde{a} &= a + bq - a_m. \end{aligned} \tag{5}$$

Applying Equation (5) and subtracting the differential equations of the closed-loop system and reference model, the error equation is obtained

$$\dot{e} + a_m e = \tilde{b}w - \tilde{a}y_p \tag{6}$$

where e defines the error between the plant output and model reference response

$$e = y_p - y_m. \tag{7}$$

Introducing a Lyapunov function

$$V(e, \tilde{a}, \tilde{b}) = e^2 + \frac{1}{\gamma_f}\tilde{b}^2 + \frac{1}{\gamma_q}\tilde{a}^2 \tag{8}$$

which is positive definite in $\mathcal{R}^3$, the space $(e, \tilde{a}, \tilde{b})$. The time derivative of $V(e, \tilde{a}, \tilde{b})$ along Equation (6) is given by

$$\dot{V}(e, \tilde{a}, \tilde{b}) = -2a_m e^2 + \frac{2}{\gamma_f}\tilde{b}\dot{\tilde{b}} + 2\tilde{b}we + \frac{2}{\gamma_q}\tilde{a}\dot{\tilde{a}} - 2\tilde{a}y_p e. \tag{9}$$

The condition $\dot{V}(e, \tilde{a}, \tilde{b}) \leqslant 0$ leads to the adaptive control laws in Equation (10)

$$\begin{aligned} \dot{f} &= -\frac{\gamma_f}{b}ew, \\ \dot{q} &= \frac{\gamma_q}{b}ey_p \end{aligned} \tag{10}$$

where time-invariance of the plant is assumed. If the sign of the so-called high-frequency gain b is known in advance Equation (10) can be rewritten as

$$\begin{aligned} &\dot{f} = -\gamma_f^* \operatorname{sign}(b)ew, \\ &\dot{q} = \gamma_q^* \operatorname{sign}(b)ey_p \end{aligned} \tag{11}$$

where γ_f^* and γ_q^* are arbitrary positive constants.

Global stability is obtained in small operation region where the process can be sufficiently described by linear model. Problems arise in the case of unmodelled dynamics and nonlinear process plants. Our main motivation was to find a simple solution for adaptive control of nonlinear processes.

2.2. DMRFAC OF NONLINEAR SYSTEMS

The model of the plant in the proposed control scheme is given in the form of simple fuzzy Takagi–Sugeno model. The basic idea of model reference adaptive control is to introduce a global stability criterion into the design procedure and to choose the adaptive control law in such a way that the requirements of the stability criterion are fulfilled [13]. The algorithm is based on direct model reference adaptive control obtained by Lyapunov criterion. The main idea of our approach is fuzzification of adaptive parameters. The parameters are fuzzified corresponding to the process input, output or state variables of the process. Direct fuzzy adaptive controller directly adjust the parameters of fuzzy controller to achieve asymptotic tracking of the model-reference input [17, 18]. It has been show that asymptotic tracking convergence is possible if approximation error is square integrable. This mean that the fuzzy basis function expansion or fuzzy modelling of the plant dynamics should be designed in a way to achieve the modelling error, i.e. error between the real plant dynamics and the model, which is square integrable. Even in the case where the approximation, i.e. the modelling or approximation error is not square integrable we have shown that it is possible to achieve the asymptotic tracking of the model-reference signal.

The main advantage of the proposed approach is simplicity together with high performance, and it has been proven that the closed-loop system using direct fuzzy adaptive controller is globally stable and the tracking error converges to the residual set which depends on fuzzification properties. The proposed approach can be implemented on a wide range of industrial processes.

The paper is focused only on problem of nonlinearity. All other typical problems which are common for adaptive system in general can be treated and solved as proposed in literature [7, 8, 19].

The algorithm of Direct Model-Reference Fuzzy Adaptive Control (DMRFAC) will be presented next.

Because of the simplicity we are assuming the first order model plant, but the approach can be easily extend to higher order.

The proposed fuzzy adaptive control system assumes the fuzzification of forward gain $\boldsymbol{f}$ and feedback gain $\boldsymbol{q}$. The choice of fuzzification variables depends on the process behaviour and is similar problem to that of structural identification in case of Takagi–Sugeno (TS) model [24]. The fuzzified gains are described by means of fuzzy numbers $\boldsymbol{f}$ and $\boldsymbol{q}$

$$\begin{aligned} \boldsymbol{f}^{\mathrm{T}} &= [f_1, f_2, \ldots, f_k], \\ \boldsymbol{q}^{\mathrm{T}} &= [q_1, q_2, \ldots, q_k] \end{aligned} \tag{12}$$

where k stands for number of fuzzy rules.

It should be emphasized that the structure of fuzzy model could be in general very complex but we are assuming, because of the simplicity, that the process under investigation can be modelled by the TS fuzzy model given in Equation (13). In this case the nonlinearity mostly depends on two variables which are the process output y_p and measured disturbance z_m.

$$\mathbf{R}^i: \quad \text{if } y_p \text{ is } \mathbf{A}_i \text{ and } z_m \text{ is } \mathbf{B}_i \quad \text{then } \dot{y}_p = -a_i y_p + b_i u, \quad i = 1, \ldots, k. \tag{13}$$

The variables in premise are those which influent mostly to the process nonlinearity, A_i, B_i are fuzzy membership functions where $i_a = 1, \ldots, n_a$ and $i_b = 1, \ldots, n_b$. The number of membership functions for the first and the second input variable defines the number of rules $k = n_a \times n_b$. The membership functions have to cover the whole operating area of the closed-loop system. Using this type of TS fuzzy model and the models of higher order in consequent part of the rule, a huge number of industrial processes can be modelled. The output of TS model is then given by the following equation

$$\dot{y}_p = \frac{\sum_{i=1}^{k} (\beta_i^*(k)(-a_i y_p + b_i u))}{\sum_{i=1}^{k} \beta_i^*(k)}. \tag{14}$$

The degree of fulfillment $\beta_i^*(k)$ is obtained using a simple algebraic product which stand for fuzzy intersection or fuzzy T-norm in general, and which replace linguistic fuzzy operator and

$$\beta_i^*(k) = T\bigl(\mu_{A_i}(y_p), \mu_{B_i}(z_m)\bigr) = \mu_{A_i}(y_p).\mu_{B_i}(z_m) \tag{15}$$

where $\mu_{A_i}(y_p)$ and $\mu_{B_i}(z_m)$ stand for degrees of fulfillment of the corresponding membership functions. The degrees of fulfillment for the whole set of rules can be written as

$$\boldsymbol{\beta}^* = \left[\beta_1^*, \beta_2^*, \ldots, \beta_k^*\right]^{\mathrm{T}} \tag{16}$$

and given in normalized form as

$$\boldsymbol{\beta} = \frac{\boldsymbol{\beta}^*}{\sum_{i=1}^{k} \beta_i^*} = [\beta_1, \beta_2, \ldots, \beta_k]^{\mathrm{T}} \tag{17}$$

resulting in equality

$$\sum_{i=1}^{k} \beta_i = 1. \tag{18}$$

Due to Equations (14) and (17) the process can be modeled in fuzzy form as

$$\dot{y}_p = -\boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{a} y_p + \boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{b} u \tag{19}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ stand for fuzzified parameters of the process which have constant elements

$$\begin{aligned} \boldsymbol{a}^{\mathrm{T}} &= [a_1, a_2, \ldots, a_k], \\ \boldsymbol{b}^{\mathrm{T}} &= [b_1, b_2, \ldots, b_k]. \end{aligned} \tag{20}$$

To develop DMRFAC algorithm the control law given in Equation (21) is assumed first,

$$u = \boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{f} w - \boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{q} y_p \tag{21}$$

where $\boldsymbol{f}$ and $\boldsymbol{q}$ are control parameters to be obtained by adaptive law. The control law given in Equation (21) is in general not applicable because the unknown high-frequency gain vector $\boldsymbol{b}$. Further development will result in an applicable control law. Implementing the control law in the basic closed loop of the control system the following differential equation is obtained

$$\dot{y}_p = -\boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{a} y_p + \boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{f} b_{\min} w - \boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{q} b_{\min} y_p + \epsilon(\boldsymbol{\beta}, \boldsymbol{b}, \boldsymbol{f}, \boldsymbol{q}). \tag{22}$$

Equation (22) is obtained under the following assumptions:

$$\begin{aligned} \left(\boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{b}\right)\left(\boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{f}\right) w &= \left(\boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{f}\right) b_{\min} w + \epsilon_f(\boldsymbol{\beta}, \boldsymbol{b}, \boldsymbol{f}, \boldsymbol{q}), \\ \left(\boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{b}\right)\left(\boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{q}\right) y_p &= \left(\boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{q}\right) b_{\min} y_p + \epsilon_q(\boldsymbol{\beta}, \boldsymbol{b}, \boldsymbol{f}, \boldsymbol{q}), \\ \epsilon(\boldsymbol{\beta}, \boldsymbol{b}, \boldsymbol{f}, \boldsymbol{q}) &= \epsilon_f(\boldsymbol{\beta}, \boldsymbol{b}, \boldsymbol{f}, \boldsymbol{q}) + \epsilon_q(\boldsymbol{\beta}, \boldsymbol{b}, \boldsymbol{f}, \boldsymbol{q}), \\ b_{\min} &= \min_i \{\boldsymbol{b}\}. \end{aligned} \tag{23}$$

In Stone–Weierstrass theorem [5] it is shown that the general form of TS fuzzy model represents a universal approximator of any nonlinear dynamic system by fuzzy basis function expansion with an arbitrary precision. For the proposed class of processes this means that the unmodelled term in Equation (22) fulfills the criterion in Equation (24).

$$|\epsilon(\boldsymbol{\beta}, \boldsymbol{b}, \boldsymbol{f}, \boldsymbol{q})| \leqslant \bar{\epsilon} \tag{24}$$

The fuzzy reference model parameters $\boldsymbol{a}_m$ and $\boldsymbol{b}_m$ can be defined as

$$\begin{aligned} \boldsymbol{a}_m &= [a_m, a_m, \dots, a_m]^{\mathrm{T}}, \\ \boldsymbol{b}_m &= [b_m, b_m, \dots, b_m]^{\mathrm{T}} \end{aligned} \tag{25}$$

and the reference model is written in the form of fuzzy model by the following equation

$$\dot{y}_m = -\boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{a}_m y_m + \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{b}_m w. \tag{26}$$

Subtracting differential equation with fuzzy parameters in Equation (26) from Equation (22) the following error model is obtained

$$\begin{aligned} \dot{e} + \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{a}_m e &= \boldsymbol{\beta}^{\mathrm{T}}(\boldsymbol{f} b_{\min} - \boldsymbol{b}_m)w - \boldsymbol{\beta}^{\mathrm{T}}(\boldsymbol{q} b_{\min} + \boldsymbol{a} - \boldsymbol{a}_m)y_p \\ &\quad + \epsilon(\boldsymbol{\beta}, \boldsymbol{b}, \boldsymbol{f}, \boldsymbol{q}). \end{aligned} \tag{27}$$

The following equations are written to simplify further derivation

$$\begin{aligned} \tilde{\boldsymbol{b}} &= \boldsymbol{f} b_{\min} - \boldsymbol{b}_m, \\ \tilde{\boldsymbol{a}} &= \boldsymbol{q} b_{\min} + \boldsymbol{a} - \boldsymbol{a}_m. \end{aligned} \tag{28}$$

Implementing expressions from Equation (28) to Equation (27) the simplified equation is given by

$$\dot{e} + \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{a}_m e = \boldsymbol{\beta}^{\mathrm{T}}\tilde{\boldsymbol{b}} w - \boldsymbol{\beta}^{\mathrm{T}}\tilde{\boldsymbol{a}} y_p + \epsilon(\boldsymbol{\beta}, \boldsymbol{b}, \boldsymbol{f}, \boldsymbol{q}). \tag{29}$$

Introducing the Lyapunov function

$$V(e, \tilde{\boldsymbol{a}}, \tilde{\boldsymbol{b}}) = e^2 + \sum_{i=1}^{k} \frac{1}{\gamma_{bi}} \tilde{b}_i^2 + \sum_{i=1}^{k} \frac{1}{\gamma_{ai}} \tilde{a}_i^2 \tag{30}$$

the time derivative of the function is given by

$$\dot{V}(e, \tilde{\boldsymbol{a}}, \tilde{\boldsymbol{b}}) = 2e\dot{e} + 2\sum_{i=1}^{k} \frac{1}{\gamma_{bi}} \dot{\tilde{b}}_i \tilde{b}_i + 2\sum_{i=1}^{k} \frac{1}{\gamma_{ai}} \dot{\tilde{a}}_i \tilde{a}_i. \tag{31}$$

By fulfilling Equation (32), $V(e, \tilde{\boldsymbol{a}}, \tilde{\boldsymbol{b}}) > 0$ in Equation (31) can be made negatively semi-definite if tracking error e exceeds boundary which is a function of unmodelled term $\epsilon(\boldsymbol{\beta}, \boldsymbol{b}, \boldsymbol{f}, \boldsymbol{q})$. This means that the closed-loop system using direct fuzzy adaptive controller is globally stable and the tracking error converges to the residual set which depends on fuzzification properties:

$$
\begin{aligned}
ew\sum_{i=1}^{k}\beta_i\tilde{b}_i + \sum_{i=1}^{k}\frac{1}{\gamma_{bi}}\dot{\tilde{b}}_i\tilde{b}_i &= 0,\\
ey_p\sum_{i=1}^{k}\beta_i\tilde{a}_i + \sum_{i=1}^{k}\frac{1}{\gamma_{bi}}\dot{\tilde{a}}_i\tilde{a}_i &= 0, \quad i = 1,\dots,k.
\end{aligned}
\tag{32}
$$

Equation (32) leads to the adaptive laws which are obtained in the following equation

$$
\begin{aligned}
\dot{\tilde{b}}_i &= -\gamma_{bi}ew\beta_i,\\
\dot{\tilde{a}}_i &= \gamma_{ai}ey_p\beta_i, \quad i = 1,\dots,k.
\end{aligned}
\tag{33}
$$

Derivation of Equation (28) results in the following equations

$$
\begin{aligned}
\dot{\tilde{\boldsymbol{b}}} &= \dot{\boldsymbol{f}}b_{\min},\\
\dot{\tilde{\boldsymbol{a}}} &= \dot{\boldsymbol{q}}b_{\min}
\end{aligned}
\tag{34}
$$

where time-invariance of the controlled system is again assumed. Assuming Equations (33) and (34) the following equations are obtained

$$
\begin{aligned}
\dot{f}_i &= -\frac{\gamma_{bi}}{b_{\min}}ew\beta_i,\\
\dot{q}_i &= \frac{\gamma_{ai}}{b_{\min}}ey_p\beta_i, \quad i = 1,\dots,k.
\end{aligned}
\tag{35}
$$

The fuzzified vectors $\boldsymbol{f}$ and $\boldsymbol{q}$ are defined as

$$
\begin{aligned}
&\boldsymbol{f} = [f_1,\dots,f_k]^{\mathrm{T}},\\
&\boldsymbol{q}^* = [q_1,\dots,q_k]^{\mathrm{T}}.
\end{aligned}
\tag{36}
$$

According to Equation (35) the fuzzified adaptive parameters f_i and q_i where $i = 1,\dots,k$ are written as

$$
\begin{aligned}
f_i &= -\gamma_{fi}\,\mathrm{sign}(b_{\min})\int_0^t ew\beta_i\,\mathrm{d}t + f_i(0), \quad \gamma_{fi} > 0,\\
q_i &= \gamma_{qi}\,\mathrm{sign}(b_{\min})\int_0^t ey_p\beta_i\,\mathrm{d}t + q_i(0), \quad \gamma_{qi} > 0,
\end{aligned}
\tag{37}
$$

where

$$
\begin{aligned}
\gamma_{fi} &= \frac{\gamma_{bi}}{|b_{\min}|},\\
\gamma_{qi} &= \frac{\gamma_{ai}}{|b_{\min}|}.
\end{aligned}
\tag{38}
$$

To obtain a stable closed-loop system, the knowledge of the signum of elements in $\boldsymbol{b}$ is necessary. This also implies that the signs of elements in $\boldsymbol{b}$ have to be equal.

2.3. STABILITY ISSUES OF DMRFAC

The fuzzy models are universal approximators of nonlinear processes [3]. The quality of approximation depends on fuzzification of the process state domain. The fuzzification means the number, type and distribution of membership functions.

It is not possible in general to find a vector $\boldsymbol{\Theta}^*$ that would permit zero tracking error in each operating point since fuzzy modelling only guarantees arbitrary small tracking errors. This means that we should always take care about the unmodelled error. In the case of unmodelled dynamics the adaptive schemes my easily go unstable. The lack of robustness is primarily due to the adaptive law which is nonlinear in general and therefore more susceptible to modelling error effect.

The lack of robustness of adaptive law in the presence of bounded disturbance can be solved by new approaches and adaptive laws which assure boundedness of all signals in the presence of plant uncertainties. These leads to new body of work referred to as robust adaptive control.

To introduce the robustness into the adaptive control scheme we convert the pure integral action of the adaptive law to a leaky integration and is therefore referred to as the leakage modification.

In the case of unmodelled dynamics the dynamical normalization is used to obtain a robust solution. The design of the normalizing signal m will guarantee that $(\eta_s/m) \in \mathcal{L}_\infty$ and $(\boldsymbol{\Psi}_f/m) \in \mathcal{L}_\infty$ where η_s stands for modelling error term. The design of dynamical normalization signal is following

$$\begin{aligned} m^2 &= 1 + n_s^2, \\ n_s^2 &= m_s + \boldsymbol{\Psi}_f^{\mathrm{T}}\boldsymbol{\Psi}_f + u^2 + y_p^2, \\ \dot{m}_s &= -\delta_0 m_s + u^2 + y_p^2, \\ m_s(0) &= 0 \end{aligned} \tag{39}$$

where $\delta_0 > 0$ should be properly chosen [9].

The idea of leakage modification is to modify the adaptive law so that the time derivative of Lyapunov function used to analyze the adaptive scheme becomes negative in the space of the parameter when these parameters exceed certain bounds. This can be done by modification of adaptive law which is presented in scalar form as

$$\begin{aligned} \dot{f}_i &= -\gamma_f \operatorname{sign}(b_i)\epsilon w\beta_i - \gamma_f|\epsilon m|\nu_0 f_i\beta_i, \quad i = 1, 2, \ldots, k, \\ \dot{q}_i &= -\gamma_q \operatorname{sign}(b_i)\epsilon y_p\beta_i - \gamma_q|\epsilon m|\nu_0 q_i\beta_i, \quad i = 1, 2, \ldots, k, \end{aligned} \tag{40}$$

or in vector form

$$\begin{aligned} \dot{\boldsymbol{f}} &= -\boldsymbol{\Gamma}_f\epsilon w\boldsymbol{\beta} - \boldsymbol{\Gamma}_f|\epsilon m|\nu_0\boldsymbol{F}\boldsymbol{\beta}, \\ \dot{\boldsymbol{q}} &= \boldsymbol{\Gamma}_q\epsilon y_p\boldsymbol{\beta} - \boldsymbol{\Gamma}_q|\epsilon m|\nu_0\boldsymbol{Q}\boldsymbol{\beta} \end{aligned} \tag{41}$$

where we are assuming that $\mathrm{sign}(b_i) = \mathrm{sign}(b)$ for all $i = 1, \ldots, k$ and

$$\begin{aligned} \boldsymbol{\Gamma}_f &= \gamma_f \,\mathrm{sign}(b)\boldsymbol{I}_{k\times k}, \\ \boldsymbol{\Gamma}_q &= \gamma_q \,\mathrm{sign}(b)\boldsymbol{I}_{k\times k} \end{aligned} \tag{42}$$

and $\boldsymbol{F}$ and $\boldsymbol{Q}$ stand for diagonal matrices where f_i, $i = 1, \ldots, k$ and q_i, $i = 1, \ldots, k$, are the diagonal elements.

The term $|\epsilon m|\nu_0$ in Equations (40) and (41) is called the leakage term. In the literature various choices [7, 10] of the leakage term are known. The best results have been obtained by the upper choice which is called ϵ-modification introduced by Narendra and Annaswamy [13]. The constants in ϵ-modification are ν_0 which is a design constant, m is normalizing signal and ϵ is the normalized error between the reference-model output y_m and process output y_p.

In the compact matrix form the adaptive law is written as follows

$$\dot{\boldsymbol{\Theta}} = \boldsymbol{\Gamma}\boldsymbol{\Psi}_f\epsilon - \boldsymbol{\Gamma}\boldsymbol{\Theta}_{\mathrm{diag}}\boldsymbol{\beta}|\epsilon m|\nu_0 \tag{43}$$

where

$$\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{\Gamma}_f & \boldsymbol{0}_{k\times k} \\ \boldsymbol{0}_{k\times k} & \boldsymbol{\Gamma}_q \end{bmatrix} \tag{44}$$

and

$$\boldsymbol{\Theta}_{\mathrm{diag}} = \begin{bmatrix} \boldsymbol{F} \\ \boldsymbol{Q} \end{bmatrix}. \tag{45}$$

The stability properties of adaptive laws with the ϵ-modification and dynamic normalization guarantee that

$$\begin{aligned} &\epsilon, \epsilon n_s, \boldsymbol{\Theta}, \dot{\boldsymbol{\Theta}} \in \mathcal{L}_\infty, \\ &\epsilon, \epsilon n_s, \dot{\boldsymbol{\Theta}} \in \mathcal{S}\left(\nu_0 + \frac{\eta_s^2}{m^2}\right) \end{aligned} \tag{46}$$

and if n_s, $\boldsymbol{\Psi}_f \in \mathcal{L}_\infty$ and $\boldsymbol{\Psi}_f$ is persistently exciting and is independent of η_s, then $\boldsymbol{\Theta}$ converges exponentially fast to the residual set [9]

$$\mathcal{D}_\epsilon = \left\{\boldsymbol{\Theta} \in \mathcal{R}^n,\ |\boldsymbol{\Theta}| \leqslant c(\nu_0 + \bar{\eta})\right\} \tag{47}$$

where $c \geqslant 0$ and $\bar{\eta} = \sup_t(\eta_s/m)$. From Equation (46) is also clear that $\epsilon, \epsilon n_s, \dot{\boldsymbol{\Theta}}$ are bounded in $\mathcal{L}_{2e}$ sense and belongs to the μ-small in the mean square sense.

3. The Model of Nonlinear Heat-Exchanger Plant

The simulation study was done for heat-exchanger plant control which together with sensors and actuators limitation represents a serious problem from the point of optimal energy consumption. The problem lies in the nonlinearity of the system behavior. The objective of our investigation, a real temperature plant, consists of: a plate heat-exchanger, a reservoir with heated water, two thermocouples and a motor driven valve. The plate heat exchanger, through which hot water from an electrically heated reservoir is continuously circulated in the counter-current flow to cold process fluid (cold water). The thermocouples are located in the inlet and outlet flows of the exchanger; both flow rates can be visually monitored. Power to the heater may be controlled by time proportioning control using the external control loop. The flow of the heating fluid can be controlled by the proportional motor driven valve. A schematic diagram of the plant is shown in Figure 1. The temperature of heated water $T_{sp}(k)$ is measured on the temperature sensor TC4 which is on the outlet of the secondary circuit, the temperature of cold water in the inlet of secondary circuit $T_{ep}(k)$ is measured on the temperature sensor TC3 and $T_{ec}(k)$ represents the temperature of hot water in the inlet of the primary circuit which is measured on the temperature sensor TC1. The primary circuit flow $F_c(k)$ is measured on optical flow sensor F2 and is defined by motor driven valve and the secondary flow $F_p(k)$ is measured on the optical flow sensor F1.

The controlled variable of our problem is the temperature in the secondary circuit T_{sp} which is manipulated with the flow F_c. The heat-exchanger is just one part of the plant, so the sensors and the actuators should also be modelled. For nonlinear systems with well-understood physical phenomena fundamental modelling is preferable. Although the physical phenomena in the case of heat-exchanger are well investigated, there are still some physical parameters which should be estimated assuming a certain structure of the process dynamics. The simplified first-principle model of heat-exchanger is described by the following differential

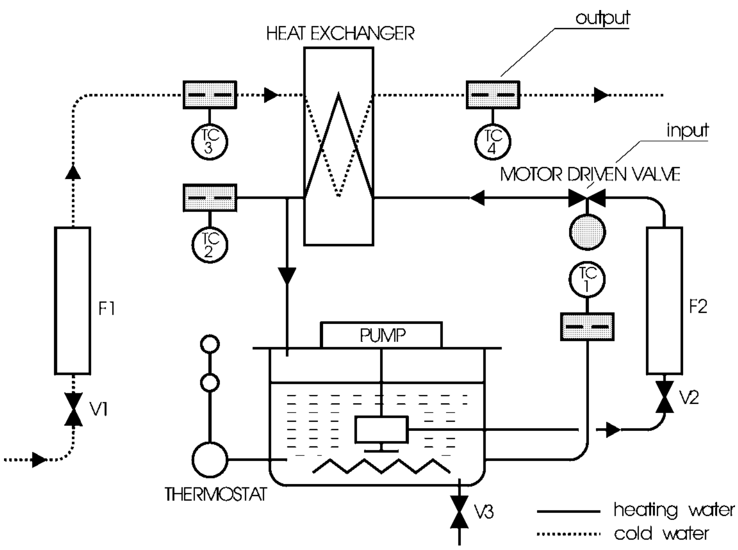

Figure 1. The heat-exchanger pilot plant.

equations

$$\tau_2(T_{sp})\dot{T}_{sp} + T_{sp} = \gamma T_{ep} + (1-\gamma)T_{ec} \tag{48}$$

where the generalized formula for γ is given in the literature [21] and can be written as

$$\gamma = \frac{1 + k_c(1/F_c)^m}{1 + k_c((1/F_c)^m + (1/F_p)^m)} \tag{49}$$

where k_c and m are unknown constants and τ_2 is an unknown function of operating point. All those parameters are unknown and should be estimated somehow.

During the operation of the heat-exchanger, some of the system variables (the flow F_p of the secondary circuit and the temperature T_{ec} at the inlet of the primary circuit of the heat-exchanger) are approximately constant. Our main goal is to control the temperature T_{sp} by changing the primary circuit flow F_p. Although the process is very complex, it could be presented as a model with approximately first order dynamics. It should be noted, however, that parasitic dynamics are also present as a consequence of actuators, sensors, heat junctions, mass flows etc. An extra parasitic pole was added to take into account for the contribution of the parasitics. That pole did not have much influence on time responses of the system, i.e. the plant was still dominantly of the first order. Measurement noise is also present in the plant. It was also added in the model of the plant.

4. Simulation Study of Model-Reference Fuzzy Adaptive Control

Our goal was to design control algorithm that would enable that the closed-loop system behaves as close to a linear reference model as possible. Two different approaches were compared: fuzzy model reference adaptive control with control law described in Equations (21) and (37), proposed in the present paper, and classical model reference adaptive control for linear plants defined by Equations (3) and (10). To make the latter robust to unmodelled dynamics, disturbances, and noise, the robust adaptive law with e_1-modification [13] was used.

The simulation study will be described next. The reference signal was periodic and piece-wise constant. The first period of the signals in DMRFAC case is shown in Figures 2, 3 and 5. In Figure 4 the trajectories of fuzzified adaptive parameters are shown. As proposed in the paper, first order reference model was chosen. Even if the plant has high order parasitics, it was forced to follow reference model, described by transfer function

$$G_m(s) = \frac{0.01}{s + 0.01}. \tag{50}$$

It is known a priori that perfect following cannot be achieved, but the results will show that this simplification is justified.

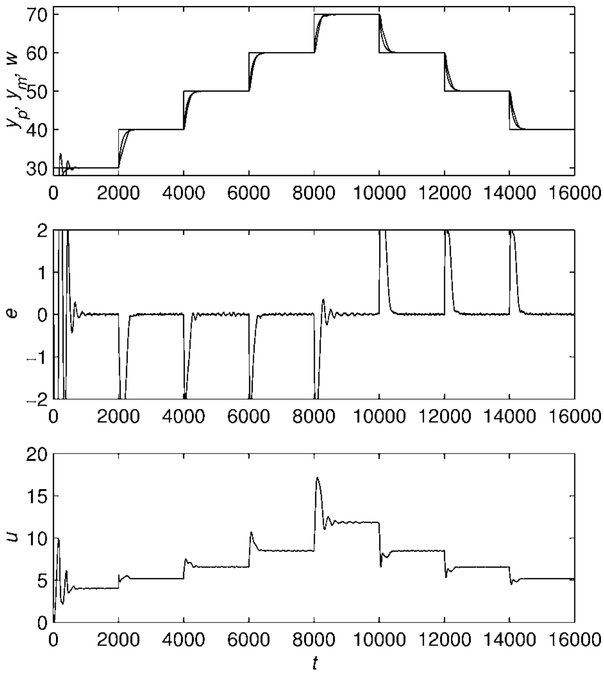

Figure 2. The first period of the signals in DMRFAC case.

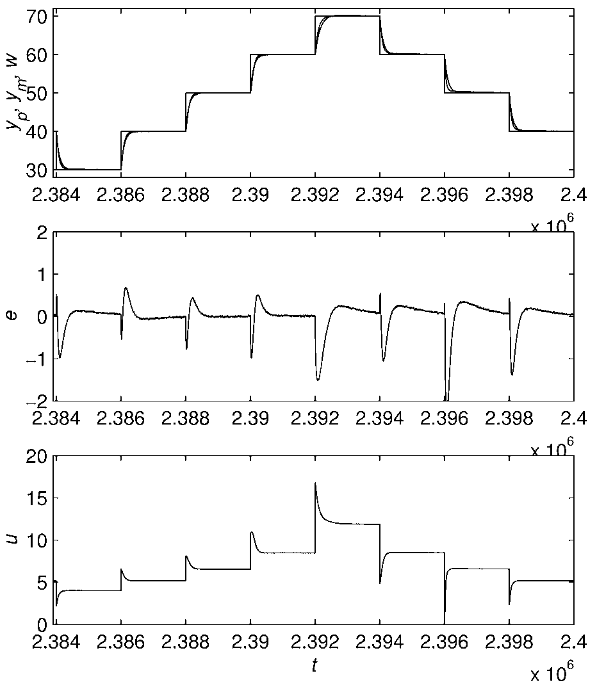

Figure 3. The last period of the signals in DMRFAC case.

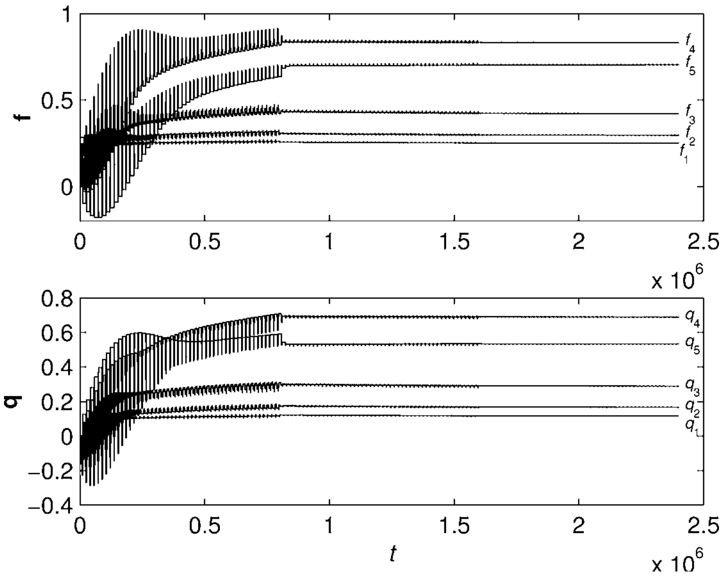

Figure 4. The trajectories of fuzzified adaptive parameters.

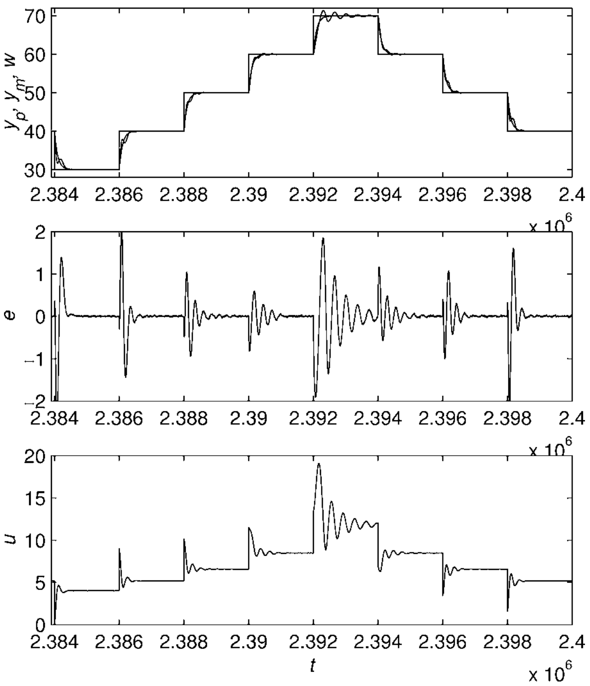

Figure 5. Last period of the signals in MRAC case.

The choice of proper adaptive gain is very important when one deals with adaptive systems. In nonlinear case this is even more obvious, since adaptive parameters do not converge to specific values. Rather, they tend to change when the operating point changes. Our approach reduces these oscillations since different parameters are estimated in different fuzzy domains. Unfortunately, fuzzy model fails to achieve perfect modelling of the plant, and the parameters still oscillate a little. These oscillations can be even further amplified by choosing adaptive gains too high. If, on the other hand, the adaptive gains are too low, the estimation is too slow, and, especially in the linear MRAC case, the parameters fail to follow rapid changes of the operating point. This problem is not so serious in the fuzzy adaptive case, since fuzzy domains have to be chosen such that the plant parameters do not differ to much in a domain. The following adaptive gains were used in our experiments: $\gamma_f = 10^{-5}$ and $\gamma_q = 10^{-5}$. The choice of the leakage parameter was not so critical in this case $\nu_0 = 1$ was used. The adaptive parameters were initialized to 0 in all cases.

First, classical MRAC was tested. It turned out that the adaptive gain was to high in the beginning and the response of the system was not acceptable. After reducing adaptive gains, the parameters converged slower, but quite good performance was achieved. Reducing adaptive gains further made the results worse. The optimal results are shown in Figure 5. In the upper part of the figure, controlled variable is shown, together with reference signal and reference model output. To see the difference more clear, tracking error is shown in the middle part of the figure. The manipulated variable is shown in the lower part of the figure. It can be seen that the results are acceptable, but the system start to oscillate when reference signal becomes large. These oscillations are also present in other operating points, they can be seen easier from the manipulated variable.

The proposed approach was also tested under the same conditions. The original adaptive gain was used in the beginning of the experiment. The estimated parameters were quite oscillatory, but that did not affect plant responses. The first period of the reference signal is shown in Figure 2. It can be seen, that the performance of the control is excellent. In the classical MRAC case, the response of the plant was very bad in the beginning. The adaptive parameters are shown in Figure 4. Huge oscillations can be seen in the beginning, but the parameters still quasy-settle. Then, the adaptive gains were reduced to obtain better convergence of the parameters. This was done twice in the experiment, and these two points can be identified as a decrease of parameter oscillations in Figure 4. The plant response at the end of the experiment is again shown in Figure 3. By comparing the signals in Figures 5 and 3, it can be said that the latter are more acceptable, since almost perfect tracking is obtained, and the unwanted oscillations in the manipulated variable are not present. One has to realize that the actuator in this case is a pump, and oscillations on the manipulated variable are not admissible. Note the tracking error when the reference signal changes (Figure 3). It cannot be suppressed by any control algorithm since it is a consequence of the fact that the plant of the second order is forced to follow reference model of the first order.

5. Conclusion

In this paper a novel fuzzy model-reference adaptive control system is introduced. It is based on Lyapunov stability criterion. The adaptive parameters of the system are fuzzified. The main goal of the proposed approach was the extension of globally stable adaptive control to nonlinear plants. The parameters are fuzzified corresponding to the process input, output or state variables of the process. The development of the novel algorithm has been tested using simulation on different nonlinear systems, including also unmodelled and unmeasured dynamics. The combination of adaptive control theory based on models obtained by fuzzy basis function expansion results in fuzzy model-reference adaptive control which provides higher adaptation ability than basic adaptive control systems. The main advantage of the proposed approach is simplicity together with high performance. In the paper the foundation of proposed algorithm are given and simulation example is shown and discussed.

Acknowledgements

The author would like to thank to BMFT-KFA Jülich, Germany and Ministry of Science, Slovenia, which funded the author's stay at TU Darmstadt by prof. Isermann, where the main part of the research was done.

References

1. Abonyi, J., Andersen, H., Nagy, L., and Szeifert, F.: Inverse fuzzy-process-model based direct adaptive control, *Math. Comput. Simul.* **51**(1/2) (1999), 119–132.
2. Berstecher, R. G., Palm, R., and Unbehauen, H. D.: An adaptive fuzzy sliding-mode controller, *IEEE Trans. Industr. Electron.* **48**(1) (2001), 18–31.
3. Castro, J.: Fuzzy logic controllers are universal approximators, *IEEE Trans. Systems Man Cybernet.* (1995), 629–635.
4. Fink, A., Fischer, M., Nelles, O., and Isermann, R.: Supervision of nonlinear adaptive controllers based on fuzzy models, *Control Engrg. Practice* **8**(10) (2000), 1093–1105.
5. Goldberg, R. R.: *Methods of Real Analysis*, Wiley, New York, 1976.
6. Hanson, M. A. and Seborg, D. E.: Adaptive nonlinear control of pH neutralization process, *IEEE Trans. Control Systems Technol.* **2**(3) (1994).
7. Ioannou, P. A. and Kokotovic, P. V.: Instability analysis and improvement of robustness of adaptive control, *Automatica* **20**(5) (1984), 583–594.
8. Ioannou, P. A. and Sun, J.: Theory and design of robust direct and indirect adaptive-control schemes, *Internat. J. Control* **47**(3) (1988), 775–813.
9. Ioannou, P. A. and Sun, J.: *Robust Adaptive Control*, Prentice-Hall, Englewood Cliffs, NJ, 1996.
10. Ioannou, P. A. and Tsakalis, K. S.: A robust direct adaptive controller, *IEEE Trans. Automat. Control* **31**(11) (1986), 1033–1043.

11. Jagannathan, S., Lewis, F. L., and Pastravanu, O.: Model reference adaptive control of nonlinear dynamical systems using multilayer neural networks, in: *Proc. of IEEE Internat. Conf. on Neural Networks*, Vol. 7, 1994, pp. 4766–4711.
12. Krstic, M., Kanellakopoulos, I., and Kokotovic, P.: *Nonlinear and Adaptive Control Design*, Wiley, New York, 1995.
13. Narendra, K. S. and Annaswamy, A. M.: *Stable Adaptive Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1989.
14. Narendra, K. S. and Annaswamy, A. M.: Robust adaptive control in presence of bounded disturbance, *IEEE Trans. Automat. Control* **31** (1985), 306–315.
15. Narendra, K. S. and Balakrishnan, J.: Adaptive control using multiple models, *IEEE Trans. Automat. Control* **42**(2) (1997), 171–187.
16. Narendra, K. S. and Parthasarathy, K.: Identification and control of dynamical systems using neural networks, *IEEE Trans. Neural Networks* **1**(1) (1990), 4–27.
17. Narendra, K. S. and Valavani, L. S.: Stable adaptive controller design – Direct control, *IEEE Trans. Automat. Control* **23** (1979), 570–583.
18. Narendra, K. S. and Valavani, L. S.: Direct and indirect model reference adaptive control, *Automatica* **15** (1979), 653–664.
19. Ortega, R. and Tang, Y.: Robustness of adaptive controllers – A survey, *Automatica* **25**(5) (1989), 651–677.
20. Passino, K. M. and Yurkovich, S.: *Fuzzy Control*, Addison-Wesly, Reading, MA, 1998.
21. Richalet, J.: Industrial application of model based predictive control, *Automatica* **29**(5) (1993), 1251–1274.
22. Rohrs, C. E., Valavani, L., Athans, M., and Stein, G.: Robustness of continuous-time adaptive control algorithms in the presence of unmodeled dynamics, *IEEE Trans. Automat. Control* **30**(9) (1985), 881–889.
23. Spooner, J. T. and Passino, K. M.: Stable adaptive control using fuzzy systems and neural networks, *IEEE Trans. Fuzzy Systems* **4**(3) (1996).
24. Takagi, T. and Sugeno, M.: Fuzzy identification of systems and its applications to modelling and control, *IEEE Trans. Systems Man Cybernet.* **15** (1985), 116–132.
25. Wang, L. X.: Stable adaptive fuzzy control of nonlinear systems, *IEEE Trans. Fuzzy Systems* **1**(2) (1993), 146–155.