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Fuzzy Predictive Functional Control in the State Space Domain

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Abstract. In the paper, a well-known predictive functional control strategy is extended to nonlinear processes. In our approach the predictive functional control is combined with a fuzzy model of the process and formulated in the state space domain. The prediction is based on a global linear model in the state space domain. The global linear model is obtained by the fuzzy model in Takagi–Sugeno form and actually represents a model with changeable parameters. A simulation of the system, which exhibits a strong nonlinear behaviour together with underdamped dynamics, has evaluated the proposed fuzzy predictive control. In the case of underdamped dynamics, the classical formulation of predictive functional control is no longer possible. That was the main reason to extend the algorithm into the state space domain. It has been shown that, in the case of nonlinear processes, the approach using the fuzzy predictive control gives very promising results.

Key words: fuzzy identification, predictive control.

1. Introduction

The predictive control has become a very important area of research in the recent years. The principle is based on the forecast of the output signal y at each sampling instant. The forecast is made implicitly or explicitly according to the model of the controlled process. In the next step the control is selected, which brings the predicted process output signal back to the reference signal to minimise the difference between the reference and the output signal. Fundamental methods which are essentially based on the principles of predictive control are Clarke's method (generalised predictive control [6]), Richalet's method (model algorithmic control and predictive functional control [14]), Cutler's method (dynamic matrix control [7]), De Keyser's method (extended prediction self-adaptive control [9]), and Ydstie's method (extended horizon adaptive control [18]).

In the paper, the predictive functional control strategy is extended to nonlinear systems. In this approach the predictive functional control is combined with the fuzzy model of the process and formulated in the state space domain. The reformulation into the state space domain is needed in the case where we are dealing with the dynamics with complex poles of the transfer function. In those cases the state space approach will lead to a simple solution. In the case of underdamped dynamics, the classical formulation of predictive functional control is no longer possible and that was also the main reason to extend the algorithm into the state space domain.

The prediction of the output variable is based on a global linear model and formulated in the state space domain. A simulation of the system, which exhibits a strong nonlinear behaviour together with the underdamped dynamics, has evaluated the proposed fuzzy predictive control. The proposed fuzzy predictive functional control algorithm shows very promising results in the case of very difficult processes, such as strongly nonlinear processes, processes with a long time delay and non-minimum phase processes. The controllers based on the prediction strategy also exhibit a remarkable robustness with respect to model mismatch and unmodeled dynamics.

The paper is organised in the following way: Section 2 deals with the concept of fuzzy identification, in Section 3, the concept of predictive control based on the fuzzy model in the state space domain is given and, finally, the simulation results of the proposed control algorithm are shown in Section 4.

2. Fuzzy Identification

Initially, the fuzzy models used for control were models of the controllers. The knowledge of experienced plant operators was used as the knowledge base in the fuzzy inference engine. The benefit of such an approach is a direct applicability of the resulting controller. The same procedure can be used to design a model of the controlled plant. However, such models have been seldom used since there is no direct mapping between the model and the controller.

In terms of classical modelling the basic approach in building fuzzy models is called theoretical modelling. The quantitative knowledge about an object to be modelled is formulated in the form of **if–then** rules. The number of the rules and their form correspond to the model structure, while the shape and the number of membership functions, the choice of fuzzy-logic operators, and the defuzzification method correspond to the fuzzy model parameters.

The fuzzy model can be treated as a universal approximator, which can approximate continuous functions to an arbitrary precision [4, 10]. In general, fuzzy logic universal approximators have several inputs and outputs. Without loss of generality only one output will be treated here. The approximators with more than one output can be treated as several approximators in parallel.

A typical Takagi–Sugeno type of rule can be written as

$$\mathbf{R}^j\colon\ \textbf{if}\ x_1 \text{ is } A_1^j \text{ and } \ldots \text{ and } x_N \text{ is } A_N^j \quad \textbf{then}\ y = f^j(x_1, \ldots, x_N), \tag{1}$$

where $x_1, \ldots, x_N$ are inputs, A_i^j are subsets of the input space, y is the output, and f^j is a function (in general, nonlinear, usually, linear).

However, the system dynamics can be modelled even by the Mamdani type [11] of rules where the consequence is symbolic (fuzzy set):

$$\mathbf{R}^j\colon\ \textbf{if}\ x_1 \text{ is } A_1^j \text{ and } \dots \text{ and } x_N \text{ is } A_N^j \quad \textbf{then}\ y \text{ is } B^j. \tag{2}$$

The fuzzy system (fuzzification–inference–defuzzification) can be treated as a nonlinear mapping between inputs and outputs.

2.1. GLOBAL LINEAR MODEL BASED ON A TS FUZZY MODEL

The described fuzzy model represents a static nonlinear mapping between input and output variables. Dynamic systems are usually modelled by feeding back delayed input and output signals. The common nonlinear model structure is a NARX (Nonlinear AutoRegressive with eXogenous input) model [15], which gives the mapping between past input–output data and the predicted output

$$\hat{y}(k+1) = F\big(y(k), y(k-1), \dots, y(k-n+1), u(k), \dots, u(k-m+1)\big), \tag{3}$$

where $y(k), y(k-1), \dots, y(k-n+1)$ and $u(k), u(k-1), \dots, u(k-m+1)$ denote the delayed process output and input signals, respectively. The fuzzy model therefore approximates the function F. Each of the fuzzy model types has its own learning, a fuzzy reasoning algorithm, and a set of free parameters. The structure identification in the fuzzy model sense means the specification of operators for logical connectives, fuzzification, inference and defuzzification algorithms. Once the structure is determined, the consequent parameters can be estimated using the least squares method.

Fuzzy modelling or identification aims at finding the set of fuzzy **if–then** rules with well defined parameters, that can describe the given I/O behaviour of the process. In the recent years many different approaches to fuzzy identification have been proposed in the literature [2, 3, 16].

In this section, TS fuzzy models are discussed. Suppose the following rule base of the fuzzy system

$$\mathbf{R}^i\colon\ \textbf{if}\ x_1 \text{ is } A_i \text{ and } \dots \text{ and } x_2 \text{ is } B_i \quad \textbf{then}\ y = f_i(x_1, x_2), \quad i = 1, \dots, K, \tag{4}$$

where x_1 and x_2 are input variables of the fuzzy system, y is an output variable, and A_i and B_i are fuzzy sets characterised by their membership functions. The **if**-parts (antecedents) of the rules describe fuzzy regions in the space of input variables, and **then**-parts (consequence) are functions of inputs, usually defined as

$$f_i(x_1, x_2) = a_i x_1 + b_i x_2 + r_i, \quad i = 1, \dots, K, \tag{5}$$

where a_i, b_i, and r_i are the consequent parameters.

We assume that the process under investigation can be modelled by the TS fuzzy model [17] of the form

$$\begin{aligned}&\mathbf{R}^i\colon\ \textbf{if}\ u(k) \text{ is } A_{i_a} \text{ and } y_p(k) \text{ is } B_{i_b} \quad \textbf{then}\\ &y_p(k+1) = a_{1i} y_p(k) + a_{2i} y_p(k-1) + b_i u(k) + r_i, \quad i = 1, \dots, K,\end{aligned} \tag{6}$$

where $u(k)$ and $y_p(k)$ are the antecedent variables and $y_p(k+1)$ is the output variable of the fuzzy system. The output variable is a linear combination of the input variables in the consequent part, which are $u(k)$ and $y_p(k)$ and $y_p(k-1)$. A_{i_a}, B_{i_b} are the fuzzy membership functions with $i_a = 1, \ldots, n_a$ and $i_b = 1, \ldots, n_b$. The numbers of membership functions for the first and the second antecedent variable define the number of rules $K = n_a \times n_b$. The membership functions have to cover the whole operating area of the closed-loop system. The output of the TS model is given by the following equation

$$\begin{aligned} y_p(k+1) &= \sum_{i=1}^{K} \big(\beta_i\big(\varphi(k)\big)\big(a_{1i} y_p(k) + a_{2i} y_p(k-1) + b_i u(k) + r_i\big)\big), \\ i &= 1, \ldots, K, \\ \varphi(k) &= \big[u(k) \quad y_p(k)\big], \end{aligned} \tag{7}$$

where $\varphi(k)$ represents the multivariate antecedent variable, which consists of process input and output signals. The normalised degree of fulfilment, $\beta_i(\varphi(k))$ is obtained by using a T-norm which is, in our case, a simple algebraic product and is given by the following equation [16]

$$\begin{aligned} \beta_i\big(\varphi(k)\big) &= \frac{T(\mu_{A_{ia}}(u(k)), \mu_{B_{ia}}(y_p(k)))}{\sum_{i=1}^{K} T(\mu_{A_{ia}}(u(k)), \mu_{B_{ia}}(y_p(k)))} \\ &= \frac{\mu_{A_{ia}}(u(k)) \cdot \mu_{B_{ia}}(y_p(k))}{\sum_{i=1}^{K} \mu_{A_{ia}}(u(k)) \cdot \mu_{B_{ia}}(y_p(k))}, \end{aligned} \tag{8}$$

where $\mu_{A_{ia}}(u(k))$ and $\mu_{B_{ia}}(y_p(k))$ stand for the membership values [2, 3, 16]. The normalised degrees of fulfilment for the whole set of rules are written in the vector form as follows

$$\boldsymbol{\beta}\big(\varphi(k)\big) = \big[\beta_1 \quad \beta_2 \quad \ldots \quad \beta_K\big]. \tag{9}$$

Due to Equations 7 and 9 the process can be modelled in the fuzzy form as

$$y_p(k+1) = \boldsymbol{\beta}^{\mathrm{T}} \mathbf{a}_1 y_p(k) + \boldsymbol{\beta}^{\mathrm{T}} \mathbf{a}_2 y_p(k-1) + \boldsymbol{\beta}^{\mathrm{T}} \mathbf{b} u(k) + \boldsymbol{\beta}^{\mathrm{T}} \mathbf{r}, \tag{10}$$

where $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{b}$, and $\mathbf{r}$ stand for fuzzified parameters of the process model. The fuzzified parameters of the model are constant and are written in the following equations

$$\begin{aligned} \mathbf{a}_1^{\mathrm{T}} &= [a_{11} \quad a_{12} \quad \ldots \quad a_{1K}], \\ \mathbf{a}_2^{\mathrm{T}} &= [a_{21} \quad a_{22} \quad \ldots \quad a_{2K}], \\ \mathbf{b}^{\mathrm{T}} &= [b_1 \quad b_2 \quad \ldots \quad b_K], \\ \mathbf{r}^{\mathrm{T}} &= [r_1 \quad r_2 \quad \ldots \quad r_K]. \end{aligned} \tag{11}$$

The affine TS fuzzy model with a common consequence structure can be expressed as a *global linear model* with input–output dependant parameters, which are given by the following equations:

$$\begin{aligned}\tilde{a}_1\big(\varphi(k)\big) &= \boldsymbol{\beta}^{\mathrm{T}}\big(\varphi(k)\big)\mathbf{a}_1,\\ \tilde{a}_2\big(\varphi(k)\big) &= \boldsymbol{\beta}^{\mathrm{T}}\big(\varphi(k)\big)\mathbf{a}_2,\\ \tilde{b}\big(\varphi(k)\big) &= \boldsymbol{\beta}^{\mathrm{T}}\big(\varphi(k)\big)\mathbf{b},\\ \tilde{r}\big(\varphi(k)\big) &= \boldsymbol{\beta}^{\mathrm{T}}\big(\varphi(k)\big)\mathbf{r}.\end{aligned} \tag{12}$$

This procedure can be viewed as an instantaneous linearization [2, 3] of the process dynamics.

The estimation of fuzzy global linear model parameters a_{1i}, a_{2i}, b_i, and r_i for $i = 1, \ldots, K$ will be given next. The algorithm is based on Equation (10), which describes the fuzzy model of the observed process. We assume the normalised degrees of fulfilment which are also time-dependent

$$\sum_{i=1}^{K} \beta_i(k) = \boldsymbol{\beta}^{\mathrm{T}}(k)\mathbf{I} = 1, \tag{13}$$

where **I** stands for the unity vector. According to the normalised degrees of fulfilment, Equation (10) can be written in the following form

$$\begin{aligned}\boldsymbol{\beta}^{\mathrm{T}}(k)\mathbf{I}y_p(k+1) &= \boldsymbol{\beta}^{\mathrm{T}}(k)\mathbf{a}_1 y_p(k) + \boldsymbol{\beta}^{\mathrm{T}}(k)\mathbf{a}_2 y_p(k-1) + \\ &\quad + \boldsymbol{\beta}^{\mathrm{T}}(k)\mathbf{b}u(k) + \boldsymbol{\beta}^{\mathrm{T}}(k)\mathbf{r}.\end{aligned} \tag{14}$$

This leads to the form of the fuzzy model described in the following equation

$$\begin{aligned}\sum_{i=1}^{K} \beta_i(k) y_p(k+1) &= \sum_{i=1}^{K}\big(\beta_i(k)a_{1i}y_p(k) + \beta_i(k)a_{2i}y_p(k-1) + \\ &\quad + \beta_i(k)b_i u(k) + \beta_i(k)r_i\big).\end{aligned} \tag{15}$$

Equation (15) can be separated into K equations, which represent the participation of a certain rule in the whole output variable of the fuzzy model. This results in the following equations:

$$\begin{aligned}\mathbf{R}^1:\ \beta_1(k)y_p(k+1) &= \beta_1(k)a_{11}y_p(k) + \beta_1(k)a_{21}y_p(k-1) + \\ &\quad + \beta_1(k)b_1u(k) + \beta_1(k)r_1,\\ \mathbf{R}^2:\ \beta_2(k)y_p(k+1) &= \beta_2(k)a_{12}y_p(k) + \beta_2(k)a_{22}y_p(k-1) + \\ &\quad + \beta_2(k)b_2u(k) + \beta_2(k)r_2,\\ &\vdots\\ \mathbf{R}^K:\ \beta_K(k)y_p(k+1) &= \beta_K(k)a_{1K}y_p(k) + \beta_K(k)a_{2K}y_p(k-1) + \\ &\quad + \beta_K(k)b_Ku(k) + \beta_K(k)r_K.\end{aligned} \tag{16}$$

To obtain the fuzzy model parameters a_{1i}, a_{2i}, b_i, and r_i for $i = 1, \dots, K$, the following form of the regressor will be used for each rule

$$\psi_i^{\mathrm{T}}(k) = \left[\beta_i(k)y_p(k) \quad \beta_i(k)y_p(k-1) \quad \beta_i(k)u(k) \quad \beta_i(k)1\right]. \tag{17}$$

Composing the regressors of a certain rule for the whole group of input–output data pairs, the regression matrix $\mathbf{\Psi}_i$ is obtained

$$\mathbf{\Psi}_i(k) = \begin{bmatrix} \psi_i^{\mathrm{T}}(1) \\ \psi_i^{\mathrm{T}}(2) \\ \vdots \\ \psi_i^{\mathrm{T}}(N) \end{bmatrix}, \tag{18}$$

where N stands for the number of data pairs. The regressor at a certain time instant k is added to the regression matrix when the following criterion is fulfilled

$$\beta_i(k) \geqslant \delta, \quad k = 1, \dots, N, \tag{19}$$

where δ stands for a small positive number. According to the criterion in Equation (19) and assuming sufficient input excitation of the process, the matrix $\mathbf{\Psi}_i$ will result in a suitable condition number which is important to obtain the fuzzy model parameters by matrix inversion.

The output variable, which corresponds to the rule $\mathbf{R}^i$ is written in the following form

$$y_p^i(k+1) = \beta_i(k)y_p(k+1) \tag{20}$$

and will be induded in an output data vector

$$\mathbf{Y}_p^i = \begin{bmatrix} \beta_i(1)y_p(1) \\ \beta_i(2)y_p(2) \\ \vdots \\ \beta_i(N)y_p(N) \end{bmatrix}. \tag{21}$$

The prediction based on the fuzzy model for the rule $\mathbf{R}^i$ in the matrix form is written as follows

$$\hat{y}_p^i(k+1) = \psi_i^{\mathrm{T}}(k)\mathbf{\Theta}_i, \tag{22}$$

where vector $\mathbf{\Theta}_i$ contains the fuzzy model parameters for the rule $\mathbf{R}^i$

$$\mathbf{\Theta}_i^{\mathrm{T}} = [a_{1i} \quad a_{2i} \quad b_i \quad r_i]. \tag{23}$$

The fuzzy model parameters for the rule $\mathbf{R}^i$ are obtained using the least squares optimisation method

$$\mathbf{\Theta}_i = \left(\mathbf{\Psi}_i^{\mathrm{T}}\mathbf{\Psi}_i\right)^{-1}\mathbf{\Psi}_i^{\mathrm{T}}\mathbf{Y}_p^i. \tag{24}$$

By calculating the fuzzy model parameters for the whole group of rules, the fuzzy model parameters are obtained as it is given in Equation (11).

The parameters of the fuzzy model are estimated on the basis of measured input–output data using the least squares optimisation method. Our approach is based on the decomposition of the data matrix $\mathbf{\Psi}$ into K submatrices $\mathbf{\Psi}_1, \mathbf{\Psi}_2, \ldots,$ $\mathbf{\Psi}_K$ and due to this decomposition the parameters of each rule are calculated separately. This leads to a better estimate of the fuzzy parameters. The variances of the estimated parameters are smaller in comparison with the classical approach given in literature [2, 3, 16, 17]. This is due to better conditioning of submatrices $\mathbf{\Psi}_1, \mathbf{\Psi}_2, \ldots, \mathbf{\Psi}_K$ in comparison to the conditioning of the whole data matrix $\mathbf{\Psi}$.

The described instantaneous linearization gives the parameters of the global linear model that depends on the multivariate antecedent vector $\varphi(k)$. In other words, the model parameters are spanned by the antecedent vector, which is defined by the fuzzy model structure. The global linear parameters of the process can be used directly in the case of adaptive and predictive control where the controllers adapt to the dynamic changes on-line.

3. Predictive Functional Control in State Space

The Model-based Predictive Control (MBPC) is a control strategy based on the explicit use of the dynamic model of the process. The process model is used to predict the future behaviour of the process output signal over a certain horizon and to evaluate control actions to minimize a certain cost function [5, 6, 7, 9, 14, 18]. The predictive control law is in the case of linear systems obtained analytically by minimizing of the following criterion,

$$J(k, u) = \sum_{j=N_1}^{N_2} \big(y_m(k+j \mid k) - y_r(k+j \mid k)\big)^2 + \lambda \sum_{j=1}^{N_u} u^2(k+j), \tag{25}$$

where $y_m(k+j \mid k)$, $y_r(k+j \mid k)$ and $u(k+j)$ stand for j-step ahead prediction of the process output signal, the reference trajectory, and the control signal, respectively. N_l and N_2 are the minimum and maximum prediction horizon, N_u stands for the control prediction horizon and λ weights the relative importance of control and output energies. MBPC is the name for several different techniques all based on the same basic principles. Originally, the algorithms have been developed for linear systems. The idea of prediction has been extended to nonlinear systems in our approach. The basic principle from Equation (25) can be implemented in the case of nonlinear systems only by use of optimisation techniques, in general. To overcome the problem of optimisation, we try to introduce the basic principles of predictive functional control (PFC) [12, 13, 14] which seems to be most appropriate. The PFC control strategy is based on the coincidence of the model output prediction and reference model output prediction at a certain horizon called the coincidence horizon. This is the main reason why we are trying to combine the PFC control strategy with nonlinear fuzzy models. The PFC algorithm can be seen as an especial example of the criterion in Equation (25), where N_1 is equal to N_2 and λ is equal to zero. In our approach, the prediction of the process model output is obtained by using the fuzzy global linear model of the process and is given in the state space domain. This formulation has certain advantages over the classical approach, especially in the case of complex poles of the global linear transfer function. In this section, the basics of predictive functional control in the state space domain (SSFPFC) are introduced.

The fundamental principles of predictive functional control are very strong and easy to understand because they are natural and they can be rapidly grasped. The state space formulation of the PFC has potential advantages in the case of at least second order systems. We assume the process model written in the form of the fuzzy global linear model. The model is of the second order given by the following difference equation

$$y_m(k+1) = \tilde{a}_1(k)y_m(k) + \tilde{a}_2(k)y_m(k-1) + \tilde{b}_1(k)u(k) + \tilde{r}(k), \tag{26}$$

where $\tilde{a}_1(k), \tilde{a}_2(k), \tilde{b}_1(k)$, and $\tilde{r}(k)$ stand for global linear parameters of the second order system which are obtained as shown in the previous section. The prediction of the process output depends on initial values, the input variable $u(k)$ and a scalar offset $\tilde{r}(k)$. In the next step Equation (26) is rewritten in the form that is suitable for formulation in the state space domain [1]

$$\begin{aligned} &y_{m1}(k+1) - \tilde{a}_1(k)y_{m1}(k) - \tilde{a}_2(k)y_{m1}(k-1) = \tilde{b}_1(k)u(k), \\ &y_{m2}(k+1) - \tilde{a}_1(k)y_{m2}(k) - \tilde{a}_2(k)y_{m2}(k-1) = \tilde{r}_1(k), \\ &y_m(k+1) = y_{m1}(k+1) + y_{m2}(k+1). \end{aligned} \tag{27}$$

According to the difference equation in Equation (27), the global linear model in the state space domain is obtained and described by the following difference matrix equations as a response to the input variable $u(k)$

$$\begin{aligned} &\mathbf{x}_{m1}(k+1) = \widetilde{\mathbf{A}}_m(k)\mathbf{x}_{m1}(k) + \widetilde{\mathbf{B}}_{m1}(k)u(k), \\ &y_{m1}(k) = \widetilde{\mathbf{C}}_{m1}\mathbf{x}_{m1}(k), \\ &\widetilde{\mathbf{A}}_m(k) = \begin{bmatrix} 0 & 1 \\ \tilde{a}_2(k) & \tilde{a}_1(k) \end{bmatrix}, \qquad \widetilde{\mathbf{B}}_{m1}(k) = \begin{bmatrix} 0 \\ \tilde{b}_1(k) \end{bmatrix}, \qquad \widetilde{C}_{m1} = [\,1 \quad 0\,] \end{aligned} \tag{28}$$

and as a response to the model residue $\tilde{r}(k)$

$$\begin{aligned} &\mathbf{x}_{m2}(k+1) = \widetilde{\mathbf{A}}_m(k)\mathbf{x}_{m2}(k) + \widetilde{\mathbf{B}}_{m2}\tilde{r}(k), \\ &y_{m2}(k) = \widetilde{\mathbf{C}}_{m2}\mathbf{x}_{m2}(k), \\ &\widetilde{\mathbf{A}}_m(k) = \begin{bmatrix} 0 & 1 \\ \tilde{a}_2(k) & \tilde{a}_1(k) \end{bmatrix}, \qquad \widetilde{\mathbf{B}}_{m2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \widetilde{\mathbf{C}}_{m2} = [\,1 \quad 0\,]. \end{aligned} \tag{29}$$

Our goal is to define the control law $u(k)$ to force the whole closed loop system to behave like it is defined by the reference model trajectory. The reference model trajectory in the state space domain is given in Equation (30):

$$\begin{aligned}\mathbf{x}_r(k+1) &= \mathbf{A}_r\mathbf{x}_r(k) + \mathbf{B}_r w(k),\\ y_r(k) &= \mathbf{C}_r\mathbf{x}_r(k),\end{aligned}\tag{30}$$

where $w(k)$ stands for the reference signal, and $y_r(k)$ for the reference model trajectory. The reference model parameters should be chosen to fulfil Equation (31) to enable the reference trajectory tracking. In other words, the gain of the reference model should be equal to one.

$$\mathbf{C}_r(\mathbf{I} - \mathbf{A}_r)^{-1}\mathbf{B}_r = 1.\tag{31}$$

In the case of fuzzy predictive functional control, a single horizon is assumed which is called the coincidence horizon H and the constant future manipulated variable $u(k+i) = u(k),\ i = 1, \dots, H$, is taken into account. At this horizon the prediction of model output is the same as the reference model trajectory. The control signal is calculated at each time instant to force the process output or the process model output to be coincidental with the reference model trajectory at a certain horizon H. That is the main feature of the PFC algorithm which allows the use of nonlinear process models. In the case of general MBPC, the nonlinear problem can be solved only by numerical optimisation due to the criterion given in Equation (25).

The H-step ahead prediction of the process model output is calculated assuming constant global process parameters over the whole prediction horizon. The prediction is given as a sum of two responses. The first one is given as the response to the control signal

$$\begin{aligned}\mathbf{x}_{m1}(k+H \mid k) &= \widetilde{\mathbf{A}}_m^H\mathbf{x}_{m1}(k) + \big(\widetilde{A}_m^H - \mathbf{I}\big)\big(\widetilde{\mathbf{A}}_m - \mathbf{I}\big)^{-1}\widetilde{\mathbf{B}}_{m1}u(k),\\ y_{m1}(k+H \mid k) &= \widetilde{\mathbf{C}}_{m1}\big(\widetilde{\mathbf{A}}_m^H\mathbf{x}_{m1}(k) + \big(\widetilde{\mathbf{A}}_m^H - \mathbf{I}\big)\big(\widetilde{\mathbf{A}}_m - \mathbf{I}\big)^{-1}\widetilde{\mathbf{B}}_{m1}u(k)\big),\end{aligned}\tag{32}$$

and the second one as the response to residue

$$\begin{aligned}\mathbf{x}_{m2}(k+H \mid k) &= \widetilde{\mathbf{A}}_m^H\mathbf{x}_{m2}(k) + \big(\widetilde{\mathbf{A}}_m^H - \mathbf{I}\big)\big(\widetilde{\mathbf{A}}_m - \mathbf{I}\big)^{-1}\widetilde{\mathbf{B}}_{m2}\tilde{r}(k),\\ y_{m2}(k+H \mid k) &= \widetilde{\mathbf{C}}_{m2}\big(\widetilde{\mathbf{A}}_m^H\mathbf{x}_{m2}(k) + \big(\widetilde{\mathbf{A}}_m^H - \mathbf{I}\big)\big(\widetilde{\mathbf{A}}_m - \mathbf{I}\big)^{-1}\widetilde{\mathbf{B}}_{m2}\tilde{r}(k)\big).\end{aligned}\tag{33}$$

The H-step ahead prediction of process model output is given as a sum of both predicted signals and is denoted in the following equation

$$y_m(k+H \mid k) = y_{m1}(k+H \mid k) + y_{m2}(k+H \mid k).\tag{34}$$

The prediction of the reference model trajectory for H steps ahead is given in the next equation

$$\begin{aligned}\mathbf{x}_r(k+H \mid k) &= \mathbf{A}_r^H\mathbf{x}_r(k) + \big(\mathbf{A}_r^H - \mathbf{I}\big)\big(\mathbf{A}_r - \mathbf{I}\big)^{-1}\mathbf{B}_r w(k),\\ y_r(k+H \mid k) &= \mathbf{C}_r\big(\mathbf{A}_r^H\mathbf{x}_r(k) + \big(\mathbf{A}_r^H - \mathbf{I}\big)\big(\mathbf{A}_r - \mathbf{I}\big)^{-1}\mathbf{B}_r w(k)\big).\end{aligned}\tag{35}$$

The main idea of the PFC is the equivalence of the process objective increment and the process model output increment at a certain horizon. The process objective increment Δ_p is defined as the difference between the predicted reference trajectory $y_r(k+H)$ and the actual process output signal $y_p(k)$

$$\Delta_p = y_r(k+H \mid k) - y_p(k). \tag{36}$$

Assuming Equation (35) the objective increment Δ_p is defined as follows

$$\Delta_p = \mathbf{C}_r \mathbf{A}_r^H \mathbf{x}_r(k) + \mathbf{C}_r\big(\mathbf{A}_r^H - \mathbf{I}\big)\big(\mathbf{A}_r - \mathbf{I}\big)^{-1}\mathbf{B}_r w(k) - y_p(k). \tag{37}$$

The process model output increment Δ_m is defined by the next equation

$$\Delta_m = y_m(k+H \mid k) - y_m(k). \tag{38}$$

According to Equation (34), the model output increment is as follows

$$\Delta_m = y_{m1}(k+H \mid k) + y_{m2}(k+H \mid k) - y_m(k). \tag{39}$$

Equation (40), which demands the equivalence of the process objective increment and the process model output increment, defines the control law of the PFC algorithm

$$\Delta_m = \Delta_p. \tag{40}$$

The control law of the PFC is obtained in explicit analytical form as

$$\begin{aligned}
u(k) &= \frac{y_r(k+H \mid k) - \widetilde{\mathbf{C}}_{m1}\widetilde{\mathbf{A}}_m^H \mathbf{x}_{m1}(k) - y_{m2}(k+H \mid k) + e(k)}{\widetilde{\mathbf{C}}_{m1}(\widetilde{\mathbf{A}}_m^H - \mathbf{I})(\widetilde{\mathbf{A}}_m - \mathbf{I})^{-1}\widetilde{\mathbf{B}}_{m1}}, \\
e(k) &= y_m(k) - y_p(k), \\
y_r(k+H \mid k) &= \mathbf{C}_r\big(\mathbf{A}_r^H \mathbf{x}_r(k) + \big(\mathbf{A}_r^H - \mathbf{I}\big)\big(\mathbf{A}_r - \mathbf{I}\big)^{-1}\mathbf{B}_r w(k)\big), \\
y_{m2}(k+H \mid k) &= \widetilde{\mathbf{C}}_{m2}\big(\widetilde{\mathbf{A}}_m^H \mathbf{x}_{m2}(k) + \big(\widetilde{\mathbf{A}}_m^H - \mathbf{I}\big)\big(\widetilde{\mathbf{A}}_m - \mathbf{I}\big)^{-1}\widetilde{\mathbf{B}}_{m2}\tilde{r}\big).
\end{aligned} \tag{41}$$

4. Simulation Example

The predictive approach discussed in the previous section has been tested by simulation. The test plant model is a magnetic positioning system that consists of an electromagnet and a mass-spring-friction system. The mass moves in a horizontal position due to the electromagnetic force produced by the electromagnet. The electromagnetic field depends on electric current through the electromagnet. The equations, which describe the dynamics of the system, are written in the following form

$$M\ddot{y} + B(y)\dot{y} + K(y)y = F_m, \qquad F_m = c\frac{i^2}{y^2}, \tag{42}$$

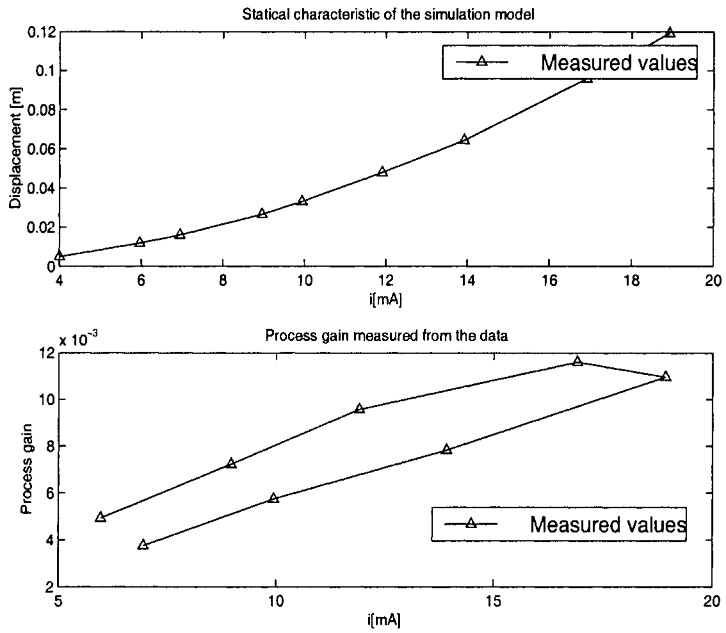

Figure 1. The plant and fuzzy model characteristics.

where y, M, $B(y)$, $K(y)$, and F_m stand for the translation displacement, the mass of moving body, the viscous-frictional coefficient which is a function of displacement, the spring coefficient which also depends on displacement, and the electromagnetic force, respectively. Look-up tables give the viscous-frictional and the spring coefficients.

The main goal of the electromechanical system is to control the position of the mass displacement y by the electric current through the electromagnet i. The system is strongly nonlinear and also has an underdamped dynamics, due to the system's coefficient. The control demand is the fastest closed-loop response without overshoot in the whole operating region. This goal is impossible to obtain using classical linear controllers due to the nonlinear process gain.

Assuming unknown relations for viscous-frictional coefficient B, the spring coefficient K and the electromagnetic coefficient c, the experimental fuzzy modelling was used to model the system dynamics. The process can be presented as a model of approximately second order dynamics, with changeable process gain as a function of the operating point.

The characteristics of the plant behaviour are shown in Figure 1, where the static map and process gain measured on data are shown. It is shown that the process gain varies significantly according to the operating point. The process dynamics is nonlinear and of second order and can be modelled by the following difference equation

$$y(k+2) = F\big(y(k), y(k+1), i(k)\big). \tag{43}$$

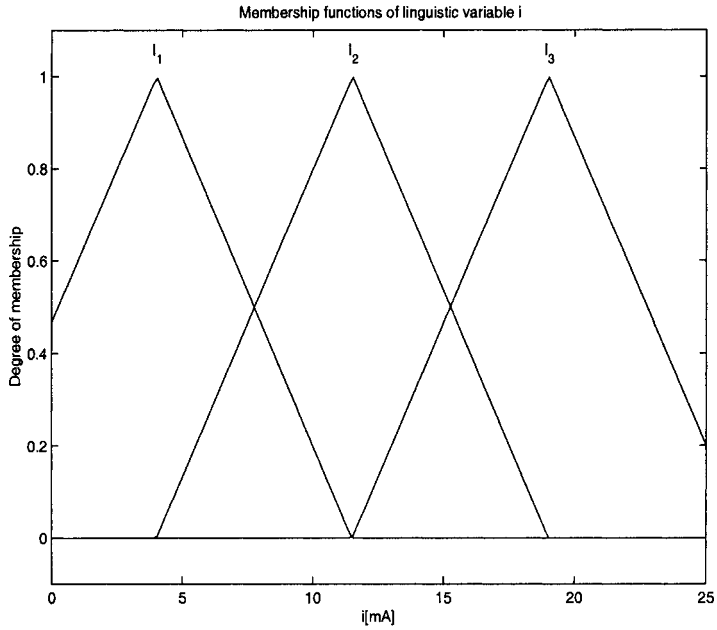

Figure 2. The linguistic variable of i.

The developed fuzzy model consists of three second order rules. The control signal i is saturated between 4 and 20 mA. The linguistic variable of i is shown in Figure 2 where we see that the operating domain is divided into three membership functions. The fuzzy parameters of the model depend on the physical variable $i(k)$, which is a manipulated variable also denoted as $u(k)$.

The fuzzy model in TS form presented by Equation (44) is obtained by the fuzzy identification method described in Section 2. The signals were sampled with sampling time $T_s = 0.1$ s.

$$\begin{aligned}
\mathbf{R}^1\!: \ \textbf{if}\ u(k)\ \text{is}\ I_1 \quad \textbf{then}\ y_p(k+1) &= 1.3076y_p(k) - 0.6771y_p(k-1) + \\
&\quad + 0.0009u(k) - 0.0019, \\
\mathbf{R}^2\!: \ \textbf{if}\ u(k)\ \text{is}\ I_2 \quad \textbf{then}\ y_p(k+1) &= 1.3106y_p(k) - 0.6801y_p(k-1) + \\
&\quad + 0.0016u(k) - 0.0019, \\
\mathbf{R}^3\!: \ \textbf{if}\ u(k)\ \text{is}\ I_3 \quad \textbf{then}\ y_p(k+1) &= 1.3160y_p(k) - 0.6761y_p(k-1) + \\
&\quad + 0.0024u(k) - 0.0019.
\end{aligned} \tag{44}$$

The validation of the fuzzy model is shown in Figure 3, where the output of the simulation fuzzy model and the output of the process are compared. It can be seen that nonlinear dynamics of the electromagnetic positioning system is sufficiently modelled.

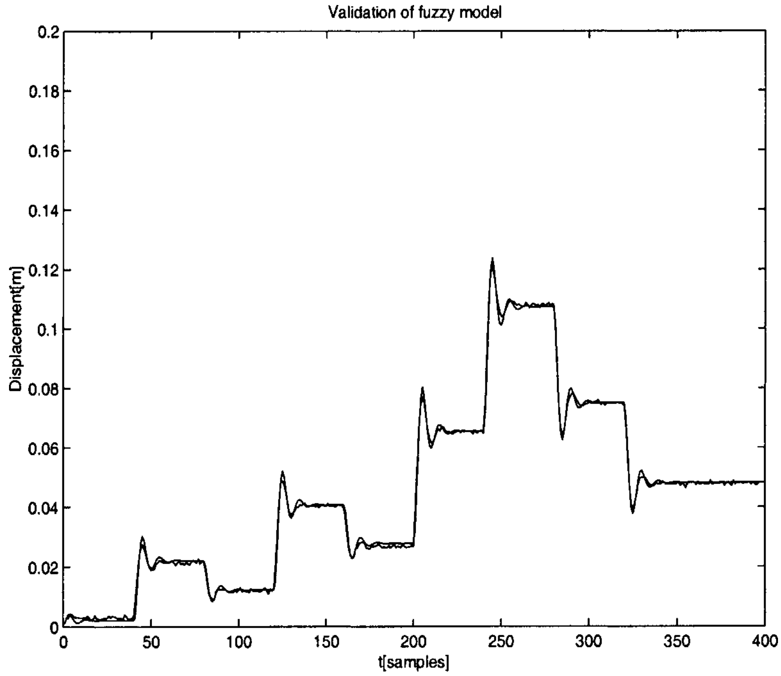

Figure 3. The validation of the fuzzy model.

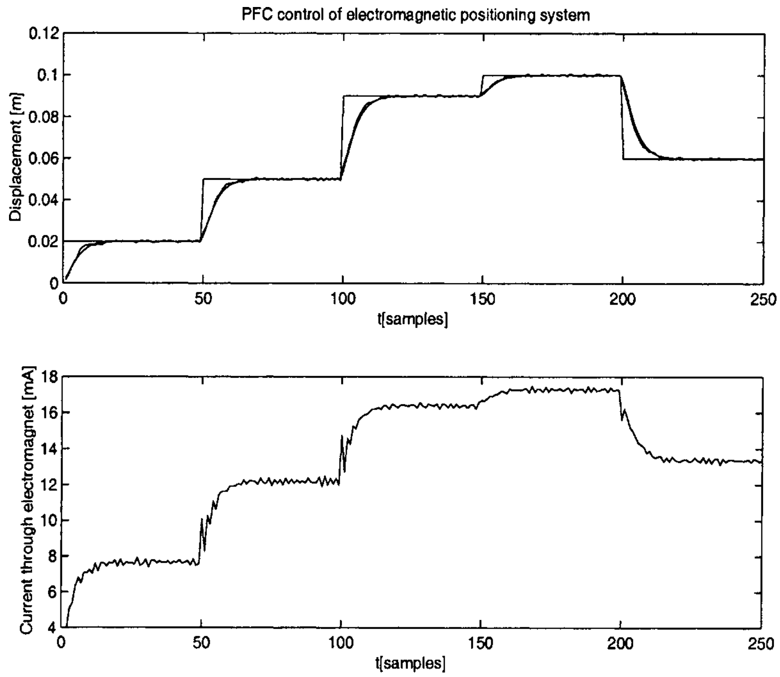

Figure 4. Output, reference-model and control signal of PFC control of the electromagnetic positioning system.

The fuzzy model in Equation (44) was used as an internal model in the predictive functional algorithm. The global linear parameters are calculated instantaneously at each sampling instant as shown in Equation (12). In respect of these parameters, the predictive functional control law is calculated according to Equation (41). In our application, the coincidence horizon has been chosen as $H = 3$ and the reference model is given by the following transfer function

$$G_r(z^{-1}) = \frac{0.09z^{-1}}{1 - 1.4z^{-1} + 0.49z^{-2}} \tag{45}$$

with the sampling time $T_s = 0.1$ s.

The process output $y(k)$ and the reference model output $y_r(k)$ are shown in Figure 4. The main demand of the control algorithm is to control the position of the mass, to have the fastest tracking response without overshoots. The problem of optimal control is difficult because of the nonlinear system behaviour. The results that have been obtained using the proposed fuzzy predictive algorithm exhibit a very good performance in both modes, in the tracking mode and disturbance rejection mode.

5. Conclusion

In this paper, the fuzzy predictive control scheme is presented. The new formulation of the fuzzy predictive scheme was motivated by the fact that the classical approach is not suitable for the systems with underdamped dynamics. Regarding the simulation experiments, it can be seen that the novel algorithm introduces a great performance in the presence of nonlinearity and unmeasured dynamics. The main advantage in comparison to the other modern techniques is simplicity together with excellent performance.

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