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Identification of dynamical systems with a robust interval fuzzy model☆

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Abstract

In this paper we present a new method of interval fuzzy model identification. The method combines a fuzzy identification methodology with some ideas from linear programming theory. On a finite set of measured data, an optimality criterion that minimizes the maximal estimation error between the data and the proposed fuzzy model output is used. The idea is then extended to modelling the optimal lower and upper bound functions that define the band that contains all the measurement values. This results in a lower and an upper fuzzy model or a fuzzy model with a set of lower and upper parameters. The model is called the interval fuzzy model (INFUMO). The method can be used when describing a family of uncertain nonlinear functions or when the systems with uncertain physical parameters are observed. We believe that the fuzzy interval model can be very efficiently used, especially in fault detection and in robust control design.

Keywords: Fuzzy model; Interval fuzzy model; Min–max optimization; Linear programming

1. Introduction

The problem of the function approximation from a finite set of measured data using an optimality criterion that minimizes the estimation error has received a great deal of attention in the scientific community, especially with the advent of neural network techniques. Continuous piecewise linear (PWL) functions have also been used for the function approximation, particularly since the introduction of the canonical expression by Chua and Deng (1988). Since then a high-level canonical piecewise linear (HLCPWL) representation of all the continuous PWL mappings defined over a simplicial partition of a domain in n-dimensional space has been introduced by Julian, Jordan, and Desages (1998) and Julian, Desages, and Agamennoni (1999). This representation is able to uniformly approximate any Lipschitz continuous function defined on a compact domain. Moreover, in contrast to neural networks, if the Lipschitz constant of the nonlinear function is known, it is possible to calculate the number of terms required to obtain a given error. An upper and lower PWL function can be evaluated to optimally describe the interval of all the possible values of the uncertain function. A salient feature of the methodology is that the approximation problem is reduced to a linear programming (LP) problem, for which efficient solution algorithms exist (Vanderbei, 1996).

The fuzzy model, in Takagi–Sugeno (TS) form, approximates the nonlinear system by smoothly interpolating affine local models (Takagi & Sugeno, 1985). Each local model contributes to the global model in a fuzzy subset of the space characterized by a membership function. In this paper we look at the development of an interval l_∞-norm function approximation methodology problem using the LP technique and the TS fuzzy-logic approach. This results in a lower and upper fuzzy model or a fuzzy model with lower and upper parameters. We call this model the interval fuzzy model (INFUMO). It is well known that the structure and shape of if-part fuzzy sets have a significant effect on the fuzzy-model approximation of continuous functions (Kosko,

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1994). Therefore, the proposed approach will exhibit an extra degree of flexibility in the domain partition as well as in the use of different membership functions compared with the HLCPWL technique.

The interval fuzzy model identification is a methodology for approximating the functions of a finite set of input and output measurements that can also be used to compress information in the case of a nonlinear function family approximation to obtain the interval of parameters that results in a band containing the whole set of measurements. This is of great importance in many technological areas, e.g., the modelling of nonlinear time-invariant systems with uncertain physical parameters, such as nonlinear circuits. The interval fuzzy models are also very efficient when used for fault detection. With fault detection the INFUMO model of a normal data set is identified and compared with the tested data. When the data of the tested system do not correspond to the tolerance band defined by lower and upper model then we can assume the faulty functioning of the corresponding system.

The paper is organized as follows: Section 2 provides the background to the fuzzy modelling; Section 3 describes the idea of fuzzy-model identification using l_∞ norm; Section 4 introduces the interval fuzzy model identification; and Section 5 presents an application to the approximation of continuous functions.

2. Nonlinear model described in fuzzy form

A typical fuzzy model (Takagi & Sugeno, 1985) is given in the form of rules

$$\mathbf{R}_j : \text{if } x_{p1} \text{ is } \mathbf{A}_{1,k_1} \text{ and } x_{p2} \text{ is } \mathbf{A}_{2,k_2} \text{ and} \dots \text{and } x_{pq} \text{ is } \mathbf{A}_{q,k_q} \text{ then } y = \phi_j(\mathbf{x}), \quad j = 1, \dots, m,$$
$$k_1 = 1, \dots, f_1, \; k_2 = 1, \dots, f_2, \dots, \; k_q = 1, \dots, f_q. \tag{1}$$

The q-element vector $\mathbf{x}_p^{\mathrm{T}} = [x_{p1}, \dots, x_{pq}]$ denotes the input or variables in premise, and the variable y is the output of the model. With each variable in premise x_{pi} $(i = 1, \dots, q)$, f_i fuzzy sets $(\mathbf{A}_{i,1}, \dots, \mathbf{A}_{i,f_i})$ are connected, and each fuzzy set $\mathbf{A}_{i,k_i}$ $(k_i = 1, \dots, f_i)$ is associated with a real-valued function $\mu_{A_{i,k_i}}(x_{pi}) : \mathbb{R} \to [0, 1]$, that produces the membership grade of the variable x_{pi} with respect to the fuzzy set $\mathbf{A}_{i,k_i}$. To make the list of fuzzy rules complete, all possible variations of fuzzy sets are given in Eq. (1), yielding the number of fuzzy rules $m = f_1 \times f_2 \times \cdots \times f_q$. The variables x_{pi} are not the only inputs of the fuzzy system. Implicitly, the n-element vector $\mathbf{x}^{\mathrm{T}} = [x_1, \dots, x_n]$ also represents an input to the system. It is usually referred to as the consequence vector. The functions $\phi_j(\cdot)$ can be arbitrary smooth functions in general, although linear or affine functions are normally used.

The system in Eq. (1) can be described in closed form if the intersection of the fuzzy sets is previously defined. The generalized form of the intersection is the so-called *triangular norm* (T-norm). In our case, the latter was chosen as an algebraic product providing the output of the fuzzy system:

$$y = \frac{\sum_{k_1=1}^{f_1} \sum_{k_2=1}^{f_2} \cdots \sum_{k_q=1}^{f_q} \mu_{A_{1,k_1}}(x_{p1}) \mu_{A_{2,k_2}}(x_{p2}) \dots \mu_{A_{q,k_q}}(x_{pq}) \phi_j(\mathbf{x})}{\sum_{k_1=1}^{f_1} \sum_{k_2=1}^{f_2} \cdots \sum_{k_q=1}^{f_q} \mu_{A_{1,k_1}}(x_{p1}) \mu_{A_{2,k_2}}(x_{p2}) \dots \mu_{A_{q,k_q}}(x_{pq})}. \tag{2}$$

It should be noted that there is a slight abuse of notation in Eq. (2), since j is not explicitly defined as a running index. From Eq. (1) it is evident that each j corresponds to the specific variation of indexes k_i, $i = 1, \dots, q$.

To simplify Eq. (2), a partition of unity is considered where the functions $\beta_j(\mathbf{x}_p)$, defined by

$$\beta_j(\mathbf{x}_p) = \frac{\mu_{A_{1,k_1}}(x_{p1}) \mu_{A_{2,k_2}}(x_{p2}) \dots \mu_{A_{q,k_q}}(x_{pq})}{\sum_{k_1=1}^{f_1} \sum_{k_2=1}^{f_2} \cdots \sum_{k_q=1}^{f_q} \mu_{A_{1,k_1}}(x_{p1}) \mu_{A_{2,k_2}}(x_{p2}) \dots \mu_{A_{q,k_q}}(x_{pq})}, \quad j = 1, \dots, m, \tag{3}$$

give information about the fulfilment of the respective fuzzy rule in the normalized form. It is obvious that $\sum_{j=1}^{m} \beta_j(\mathbf{x}_p) = 1$ irrespective of $\mathbf{x}_p$ as long as the denominator of $\beta_j(\mathbf{x}_p)$ is not equal to zero (this can be easily prevented by stretching the membership functions over the whole potential area of $\mathbf{x}_p$). Combining Eqs. (2) and (3) and changing the summation over k_i to a summation over j we arrive at the following equation:

$$y = \sum_{j=1}^{m} \beta_j(\mathbf{x}_p) \phi_j(\mathbf{x}). \tag{4}$$

From Eq. (4) it is evident that the output of a fuzzy system is a function of the premise vector $\mathbf{x}_p$ (q-dimensional) and the consequence vector $\mathbf{x}$ (n-dimensional). The dimension of the input space may be lower than $(q + n)$ since it is very common to have the same variables present in vectors $\mathbf{x}_p$ and $\mathbf{x}$. Vector $\mathbf{z}$ (d-dimensional) is composed of the elements of $\mathbf{x}_p$, and those of $\mathbf{x}$ that are not present in $\mathbf{x}_p$.

Very often, the output value is defined as a linear combination of consequence states

$$\phi_j(\mathbf{x}) = \boldsymbol{\theta}_j^{\mathrm{T}} \mathbf{x}, \quad j = 1, \dots, m, \quad \boldsymbol{\theta}_j^{\mathrm{T}} = [\theta_{j1}, \dots, \theta_{jn}]. \tag{5}$$

If the TS model of the 0th order is chosen, $\phi_j(\mathbf{x}) = \theta_{j0}$, and in the case of the first-order model, the consequent is $\phi_j(\mathbf{x}) = \theta_{j0} + \boldsymbol{\theta}_j^{\mathrm{T}} \mathbf{x}$. Both cases can be treated with model

(5) by adding 1 to the vector $\mathbf{x}$ and augmenting vector $\boldsymbol{\theta}$ with θ_{j0}. To simplify the notation, only the model in Eq. (5) will be treated in the rest of the paper. If the matrix of the coefficients for the whole set of rules is written as $\boldsymbol{\Theta}^{\mathrm{T}}=[\boldsymbol{\theta}_1,\ldots,\boldsymbol{\theta}_m]$, and the vector of membership values as $\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_p)=[\beta^1(\mathbf{x}_p),\ldots,\beta^m(\mathbf{x}_p)]$, then Eq. (4) can be rewritten in the matrix form

$$y=\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_p)\boldsymbol{\Theta}\mathbf{x}. \tag{6}$$

The fuzzy model in the form given in Eq. (6) is referred to as the affine TS model and can be used to approximate any arbitrary function that maps the compact set $\mathbf{C}\subset\mathbb{R}^d$ to $\mathbb{R}$ with any desired degree of accuracy (Kosko, 1994; Ying, 1997; Wang & Mendel, 1992). The generality can be proven with the Stone–Weierstrass theorem (Goldberg, 1976) which suggest that any continuous function can be approximated by a fuzzy basis function expansion (Lin, 1997).

3. Fuzzy model identification using l_∞ norm

In this section we discuss an approach to the model parameter estimation where the l_∞ norm is used as the criterion for the measure of the modelling error. We assume a set of premise vectors $\mathbf{X_p}=\{\mathbf{x}_{p1},\mathbf{x}_{p2},\ldots,\mathbf{x}_{pN}\}$ and a set of antecedent (or consequence) vectors $\mathbf{X}=\{\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_N\}$, from which a set $\mathbf{Z}=\{\mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_N\}$ can be constructed that represents the input measurement data, collected from the compact set $\mathbf{S}\subset\mathbb{R}^d$. A set of corresponding outputs is also defined as $\mathbf{Y}=\{y_1,y_2,\ldots,y_N\}$. The measurements satisfy the nonlinear equation of the system

$$y_i=g(\mathbf{z}_i),\quad i=1,\ldots,N. \tag{7}$$

According to the Stone–Weierstrass theorem , for any given real continuous function g on a compact set $\mathbf{U}\subset\mathbb{R}^d$ and arbitrary $\varepsilon>0$, there exist a fuzzy system f such that

$$\max_{\mathbf{z}_i\in\mathbf{Z}}|f(\mathbf{z}_i)-g(\mathbf{z}_i)|<\varepsilon,\quad \forall i. \tag{8}$$

This implies the approximation of any given real continuous function with a fuzzy function from class $\mathscr{F}^d$ defined in Eq. (6). However, it has to be pointed out that lower values of ε imply higher values of m that satisfy Eq. (8). In the case of the approximation, the error between the measured values and the fuzzy function outputs can be defined as

$$e_i=y_i-f(\mathbf{x}_i),\quad i=1,\ldots,N. \tag{9}$$

To estimate the optimal parameters of the proposed fuzzy function the minimization of the maximum modelling error

$$\max_{z_i\in\mathbf{Z}}|y_i-f(\mathbf{z}_i)| \tag{10}$$

over the whole input set $\mathbf{Z}$ is performed. This is the *min–max* optimization method. In the case of the TS model in Eq. (6), the minimization of the expression in Eq. (10) can be performed in two steps. The first problem is how to minimize the error with respect to $\mathbf{x}_p$. The answer lies in the proper arrangement of membership functions. This is a well-known problem in fuzzy systems. It can be overcome with a cluster analysis (Bezdek, Coray, Gunderson, & Watson, 1981a,b) or other approaches. The details will not be discussed in this paper. By having the membership functions defined, the structure of the model is known and only the parameters $\boldsymbol{\Theta}$ are to be defined by the *min–max* optimization

$$\boldsymbol{\Theta}=\arg\min_{\boldsymbol{\Theta}}\max_{\mathbf{z}_i\in\mathbf{Z}}|y_i-\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_{pi})\boldsymbol{\Theta}\mathbf{x}_i|. \tag{11}$$

Lemma 1. *The min–max optimization problem can be solved as the linear programming problem of minimizing λ, subject to the inequalities*

$$y_i-\sum_{j=1}^{m}\beta_j(\mathbf{x}_{pi})\boldsymbol{\theta}_j^{\mathrm{T}}\mathbf{x}_i\leqslant\lambda,\quad i=1,2,\ldots,N,$$
$$-y_i+\sum_{j=1}^{m}\beta_j(\mathbf{x}_{pi})\boldsymbol{\theta}_j^{\mathrm{T}}\mathbf{x}_i\leqslant\lambda,\quad i=1,2,\ldots,N,\quad \lambda\geqslant 0 \tag{12}$$

on the parameter $\boldsymbol{\theta}_j$ $(j=1,\ldots,m)$. The resulting λ stands for the maximum approximation error.

Proof. If we define

$$\lambda=\max_{\mathbf{z}_i\in\mathbf{Z}}\left|y_i-\sum_{j=1}^{m}\beta_j(\mathbf{x}_{pi})\boldsymbol{\theta}_j^{\mathrm{T}}\mathbf{x}_i\right| \tag{13}$$

and take into account that $\mathbf{z}_i$ encapsulates the information in vectors $\mathbf{x}_{pi}$ and $\mathbf{x}_i$, this directly implies the following system of inequalities:

$$\left|y_i-\sum_{j=1}^{m}\beta^j\boldsymbol{\theta}_j^{\mathrm{T}}\mathbf{x}_i\right|\leqslant\lambda,\quad i=1,2,\ldots,N \tag{14}$$

which can then be written in the form (12). This concludes the proof of Lemma 1, and the optimization problem from (11) can be stated as the minimization of λ subject to (12). □

The idea of an approximation can be interpreted as the most representative fuzzy function to describe the domain of outputs $\mathbf{Y}$ as a function of inputs $\mathbf{Z}$. This problem can also be viewed as a problem of data reduction, which often appears in identification problems with large data sets.

4. Interval fuzzy model identification

In the case of an uncertain nonlinear function, which can be defined as a member of the family of functions

$$\mathscr{G}=\{g:\mathbf{S}\to\mathbb{R}^1|g(\mathbf{z})=g_{\mathrm{nom}}(\mathbf{z})+\Delta g(\mathbf{z})\}, \tag{15}$$

where $g_{\rm nom}$ stands for the nominal function, the uncertainty Δg satisfies $\sup_{\mathbf{z}\in\mathbf{S}}|\Delta g(\mathbf{z})|\leqslant c,\ \ c\in\mathbb{R}$.

Let us consider a function $g\in\mathscr{G}$ and corresponding set of measured output values $\mathbf{Y}=\{y_1,\ldots,y_N\}$ over the set of inputs $\mathbf{Z}$, i.e., $y_i=g(\mathbf{z}_i)$, $g\in\mathscr{G}$, $\mathbf{z}_i\in\mathbf{S}$, $i=1,\ldots,N$.

The idea of robust interval fuzzy modelling is to find a lower fuzzy function $\underline{f}$ and an upper fuzzy function $\bar{f}$ satisfying

$$\underline{f}(\mathbf{z}_i)\leqslant g(\mathbf{z}_i)\leqslant \bar{f}(\mathbf{z}_i),\quad \forall \mathbf{z}_i\in\mathbf{S}. \tag{16}$$

In this sense, a function from class $\mathscr{G}$ can always be found in the band defined by the upper and the lower fuzzy function. The main request in defining the band is that it is as narrow as possible according to the proposed constraints. The problem has been treated in the literature using the PWL function approximation (Julian et al., 1998). Our approach using the fuzzy function approximation can be viewed as a generalization of the PWL approach and gives a better approximation, or at least a much narrower approximation band.

The upper and the lower fuzzy functions, respectively, can be found by solving the following optimization problems:

$$\min_{\underline{f}}\max_{z_i\in\mathbf{Z}}|y_i-\underline{f}(\mathbf{z}_i)|\ \text{subject to}\ y_i-\underline{f}(\mathbf{z}_i)\geqslant 0,\quad \forall i, \tag{17}$$

$$\min_{\bar{f}}\max_{z_i\in\mathbf{Z}}|y_i-\bar{f}(\mathbf{z}_i)|\ \text{subject to}\ y_i-\bar{f}(\mathbf{z}_i)\leqslant 0,\quad \forall i. \tag{18}$$

The solutions to both problems can be found by linear programming, because both problems can be viewed as linear programming problems, as is stated in the following lemma. First, we have to define a lower and an upper fuzzy function as $\underline{f}(\mathbf{z})=\boldsymbol{\beta}^{\rm T}(\mathbf{x}_p)\underline{\boldsymbol{\Theta}}\mathbf{x}$ and $\bar{f}(\mathbf{z})=\boldsymbol{\beta}^{\rm T}(\mathbf{x}_p)\bar{\boldsymbol{\Theta}}\mathbf{x}$.

Lemma 2. *The min–max optimization problems in Eqs.* (17) *and* (18) *can be solved as the linear programming problems of minimizing λ_1 and λ_2, subject to the inequalities*

$$y_i-\sum_{j=1}^{m}\beta_j(\mathbf{x}_{pi})\underline{\boldsymbol{\theta}}_j^{\rm T}\mathbf{x}_i\leqslant\lambda_1,\quad i=1,\ldots,N,$$

$$y_i-\sum_{j=1}^{m}\beta_j(\mathbf{x}_{pi})\bar{\boldsymbol{\theta}}_j^{\rm T}\mathbf{x}_i\geqslant 0,\quad i=1,\ldots,N,\quad \lambda_1\geqslant 0 \tag{19}$$

and

$$-y_i+\sum_{j=1}^{m}\beta_j(\mathbf{x}_{pi})\bar{\boldsymbol{\theta}}_j^{\rm T}\mathbf{x}_i\leqslant\lambda_2,\quad i=1,\ldots,N$$

$$y_i-\sum_{j=1}^{m}\beta_j(\mathbf{x}_{pi})\underline{\boldsymbol{\theta}}_j^{\rm T}\mathbf{x}_i\leqslant 0,\quad i=1,\ldots,N,\quad \lambda_2\geqslant 0 \tag{20}$$

on the parameters $\underline{\boldsymbol{\theta}}_j$, $\bar{\boldsymbol{\theta}}_j$, $j=1,\ldots,m$, and λ_1 and λ_2 that stand for the maximum approximation errors of both approximation functions.

Proof. The proof can be directly inferred from Lemma 1. □

The interval fuzzy modelling can be used efficiently in the case of fault detection where the data set of normal operating systems is modelled by INFUMO to obtain the band of normal functioning. During operations this band is calculated on-line and it is checked if a measurement corresponds to the normal functioning band or not. If the measurement violates the tolerance band, one can assume that a malfunction might have occurred. The proposed model can also be used for the case of robust control design as described in Ackermann (1993).

5. Interval fuzzy model of simplified car dynamics

The interval fuzzy modelling is presented for a nonlinear time-invariant system with uncertain physical parameters. These parameters are given as the interval between the minimal value and the maximal value, $\tilde{a}\in[\underline{a},\bar{a}]$ (Ackermann, 1993). The system that is observed is a simplified car dynamics with uncertain parameters of the engine force. The mathematical model that describes the dynamics is

$$\dot{v}=\frac{f_{\rm e}(u,v)-f_d}{m},$$

$$f_{\rm e}(u,v)=\tilde{K}_{\rm e}(1+a_1u)\times(1+\arctan(\tilde{a}_2u^2+a_3v+a_4)), \tag{21}$$

where v stands for the velocity of the car in m/s, u is the position of the throttle in the range [0, 1], m stands for the mass of the car and is equal to 1000 kg, $f_{\rm e}(u,v)$ is the force of the engine and f_d is the resistance force and is approximated by a constant of 1000 N. Some of the engine parameters are uncertain and vary due to the operating conditions. The values of the constants in the model are as follows: $a_1=3$, $\tilde{a}_2\in[4.2, 7.8]$, $a_3=-0.35$, $a_4=1.2$ and $\tilde{K}_{\rm e}\in[600, 900]$ N. The characteristic of the engine for a choice of uncertain parameters $K_{\rm e}=750$ N and $a_2=6$ is shown in Fig. 1. Fig. 1 shows that the characteristic of the engine is highly nonlinear.

This model, with its uncertain parameters, is used to obtain the data set for the identification of the interval fuzzy model. Five different simulated responses of the car's dynamics (a nominal one and four combinations of the maximal and minimal values of both interval parameters) to the same input signal and different values of the uncertain parameters are obtained and shown in Fig. 2. The first half of the data set was used to estimate the parameters of the INFUMO and the second half was used to validate the INFUMO. The observed time responses of the car exhibit the nonlinear dynamics of the first order. The partitioning of the space into fuzzy partitions and the number of fuzzy partitions is done using fuzzy c-means clustering algorithm as proposed in Babuška (1998), and Bezdek et al.(1981a,b) and is shown in Fig. 3. The lower and the upper bound of the whole

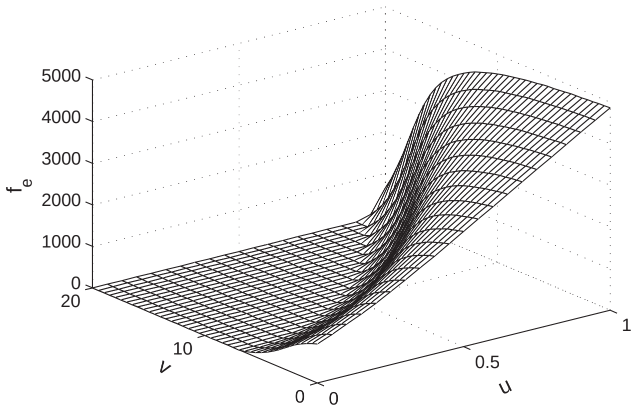

Fig. 1. The static mapping of the engine.

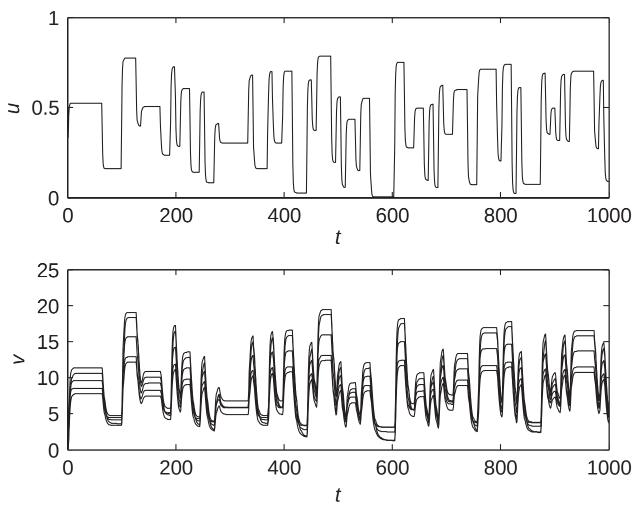

Fig. 2. The set of the modelled data—the car dynamics with uncertain parameters.

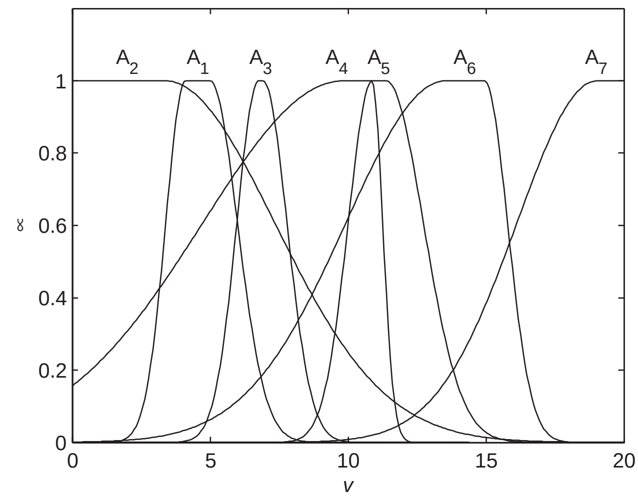

Fig. 3. The resulting membership functions obtained by *c*-means clustering algorithm.

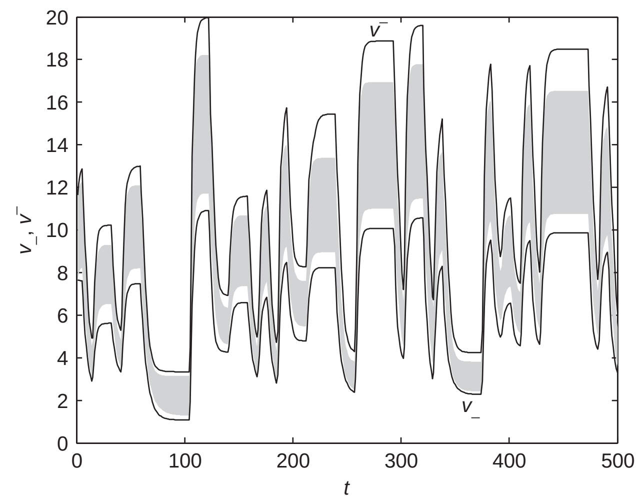

Fig. 4. The validation of INFUMO model where the grey area represents the data set of the whole family of responses.

data set can be obtained with the INFUMO of the following affine form, where the operating domain is divided into seven membership functions that are the same for the lower and upper bounds:

$$\underline{\mathbf{R}}_j : \text{if } v(k) \text{ is } \mathbf{A}_j \text{ then } \underline{v}(k+1) = \underline{a}_j \underline{v}(k) + \underline{b}_j u(k) + \underline{r}_j, \quad j = 1, \ldots, 7, \tag{22}$$

$$\bar{\mathbf{R}}_j : \text{if } v(k) \text{ is } \mathbf{A}_j \text{ then } \bar{v}(k+1) = \bar{a}_j \bar{v}(k) + \bar{b}_j u(k) + \bar{r}_j, \quad j = 1, \ldots, 7. \tag{23}$$

The final structure of the INFUMO is written in the following form with interval parameters (Ackermann, 1993):

$$\tilde{\mathbf{R}}_j : \text{ if } \tilde{v}(k) \text{ is } \mathbf{A}_j \text{ then } \tilde{v}(k+1) = \tilde{a}_j \tilde{v}(k) + \tilde{b}_j u(k) + \tilde{r}_j, \tag{24}$$

$$\tilde{a}_j = [\underline{a}_j, \bar{a}_j], \quad \tilde{b}_j = [\underline{b}_j, \bar{b}_j], \quad \tilde{r}_j = [\underline{r}_j, \bar{r}_j], \quad j = 1, \ldots, 7. \tag{25}$$

The validation of the INFUMO is shown in Fig. 4. The grey area represents the band of measured data set while the lines show the lower and upper INFUMO model responses. Fig. 5 shows the difference between INFUMO responses and the measured data set bounds. Note that $\bar{v} - \sup\, v$ is always positive and $\underline{v} - \inf\, v$ is always negative, which shows that the measured data set actually lies within the INFUMO band. As shown in Section 2, the proposed approach is not limited to only one variable in premise. The addition of an extra variable into premise would result in better approximation if this variable influences the nonlinearity of the system.

6. Conclusion

A new method of interval fuzzy model identification has been proposed that is applicable when a finite set of mea-

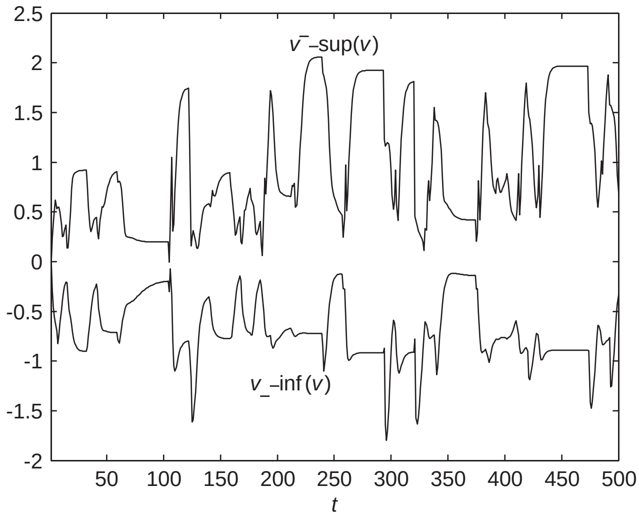

Fig. 5. The upper and the lower bound modelling error.

surement data are available. The method combines a fuzzy identification methodology with some ideas from linear programming theory. The idea is then extended to the modelling of the optimal lower and upper bound functions that define the band that contains all the measurement values. This results in the lower and upper fuzzy models or the interval fuzzy model (INFUMO). The INFUMO is of great importance in the case of families of functions where the parameters of the observed system vary in certain intervals. This approach can also be used in data mining to compress the information or in robust system identification, which can be of great importance in fault detection.

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